RI CORE STANDARDS FOR MATHEMATICS APPENDIX

Glossary, Tables, and Illustration

RIDE Rhode Island Department of Education

Glossary

A

Absolute value. The absolute value of a real number is its (non-negative) distance from 0 on a number line.
Addition and subtraction within 5, 10, 20, 100, or 1,000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0–5, 0–10, 0–20, or 0–100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.
Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and –3/4 are additive inverses of one another because 3/4 + (–3/4) = (–3/4) + ¾ = 0.
Algorithm/Standard Algorithm: See Table 6.

> **Algorithm.** A procedure for solving a mathematical problem in a finite number of steps that frequently involves repetition of an operation.
> **Standard algorithm.** A step-by-step approach to calculating, decided by *societal convention*, developed for efficiency. Flexible and fluent use of standard algorithms requires conceptual understanding.

Associative property of addition. See Table 3.
Associative property of multiplication. See Table 3.
Assumption. A fact or statement (as a proposition, axiom, postulate, or notion) taken for granted.
Attribute. A common feature of a set.

B

Benchmark fraction. A common fraction against which other fractions can be measured (e.g., ¼, ½, 2/3, 5/5).
Binomial Theorem. A theorem that gives the polynomial expansion for any whole-number power of a binomial. For powers greater than or equal to zero.
Bivariate data. Pairs of linked numerical observations. *Example: a list of heights and weights for each player on a football team.*
Box plot. A graphic method that shows the distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

C

Cardinality. An understanding of how numbers are ordered, and how to count accurately, matching a number name to the quantity counted.
Cavalieri's Principle. A method, with formula given below, of finding the volume of any solid for which cross- sections by parallel planes have equal areas. This includes, but is not limited to, cylinders and prisms. Formula: Volume = Bh, where B is the area of a cross-section and h is the height of the solid.
Coefficient. A number or variable which is a factor of a term. For example, *x* is the coefficient in the expression $x(a + b)$ and 3 is the coefficient in the term $3y$.
Commutative property. See Table 3.
Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).
Complex number. A number that can be written as the sum or difference of a real number and an imaginary number. See Illustration 1.
Complex plane. The coordinate plane used to graph complex numbers.
Compose. To put numbers or geometric figures together strategically and purposefully, typically to simplify calculation or to recognize properties
Composite number. A whole number that has more than two factors.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also: algorithm; computation strategy.*
Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also: computation algorithm.*
Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).
Conjugate. The result of writing the sum of two terms as a difference, or vice versa. For example, the conjugate of $x - 2$ is $x + 2$.
Coordinate plane. A plane in which a point is represented using two coordinates that determine the precise location of the point. In the Cartesian plane, two perpendicular number lines are used to determine the locations of points. In the polar coordinate plane, points are determined by their distance along a ray through that point and the origin, and the angle that ray makes with a pre-determined horizontal axis.
Cosine. A trigonometric function that for an acute angle is the ratio between a leg adjacent to the angle when the angle is considered part of a right triangle and the hypotenuse.
Counting number. A number used in counting objects, i.e. a number from the set 1,2,3,4, 5, . . . See Illustration 1.
Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have eight books and three more books are added to the top, it is not necessary to count the stack all over again; one can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

D

Decimal expansion. Writing a rational number as a decimal.
Decimal fraction. A fraction (as 0.25 = 25⁄100 or 0.025 = 25⁄1000) or mixed number (as 3.025 = 3 25⁄1000) in which the denominator is a power of ten, usually expressed by the use of the decimal point.
Decimal number. Any real number expressed in base ten notation.
Decompose. To take numbers or geometric figures apart strategically and purposefully, typically to simplify calculation or to recognize properties.
Digit. Digits are the numerals 0-9 found in all numbers. 176 is a 3-digit number featuring the digits 1, 7, and 6.
Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.
Directrix. A parabola is the collection of all points in the plane that are the same distance from a fixed point, called the focus (F), as they are from a fixed line, called the directrix.
Dot plot. *See: line plot.*

E

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. *For example,* 643 = 600 + 40 + 3.
Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.
Exponent. The number that indicates how many times the base is used as a factor, e.g., in $4^3 = 4 \times 4 \times 4 = 64$, the exponent is 3, indicating that 4 is repeated as a factor three times.
Exponential function. A function of the form $y = a \bullet b^x$ where $a > 0$ and either $0 < b < 1$ or $b > 1$. The variables do not have to be x and y. *For example,* $A = 3.2 \bullet (1.02)^t$ *is an exponential function.*

Expression. A mathematical phrase that combines operations, numbers, and/or variables (e.g., $3^2 \div a$).

F

Fibonacci sequence. The sequence of numbers beginning with 1, 1, in which each number that follows is the sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144. . .
First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[1] *See also: median, third quartile, interquartile range*
Fluency. Using efficient, flexible and accurate methods of computing. See Table 7.
Fraction. A number expressible in the form a/b where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a nonnegative number.) *See also: rational number.*
Function. A mathematical relation for which each element of the domain corresponds to exactly one element of the range.
Function notation. A notation that describes a function. For a function f, when x is a member of the domain, the symbol $f(x)$ denotes the corresponding member of the range (e.g., $f(x) = x + 3$).
Fundamental Theorem of Algebra. The theorem that establishes that, using complex numbers, all polynomials can be factored into a product of linear terms. A generalization of the theorem asserts that any polynomial of degree *n* has exactly *n* zeros, counting multiplicity.

G

Geometric sequence (progression). An ordered list of numbers that has a common ratio between consecutive terms, e.g., 2, 6, 18, 54, . . .

H

Histogram. A graph that uses bars that have equal ranges of values.

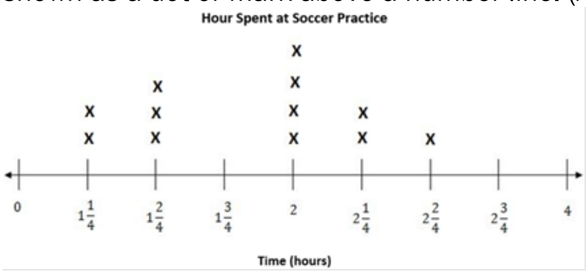
I

Identity property of 0. See Table 3.
Imaginary number. Any number of the form *bi*, where *b* is a nonzero real number and *i* is the square root of –1. See Illustration 1.
Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. All positive and negative whole numbers and zero.
Interquartile range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also: first quartile, third quartile.*
Inverse function. A function obtained by expressing the dependent variable of one function as the independent variable of another; that is the inverse of $y = f(x)$ is $x = f^{-1}(y)$.
Irrational number. A number that cannot be expressed as a quotient of two integers, e.g., $\sqrt{2}$ It can be shown that a number is irrational if and only if it cannot be written as a repeating or terminating decimal.

[1]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

L

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. (Also known as a dot plot.)



Linear association. Two variables have a linear association if a scatter plot of the data can be well approximated by a line.
Linear equation. Any equation that can be written in the form $Ax + By + C = 0$ where A and B cannot both be 0. The graph of such an equation is a line.
Linear function. A function with an equation of the form $y = mx + b$, where m and b are constants.
Logarithm. The exponent that indicates the power to which a base number is raised to produce a given number. *For example, the logarithm of 100 to the base 10 is 2.*
Logarithmic function. Any function in which an independent variable appears in the form of a logarithm; they are the inverse functions of exponential functions.

M

Matrix (pl. matrices). A rectangular array of numbers or variables.
Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[2] *Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.*
Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.*
Measure of variability. A determination of how much the performance of a group deviates from the mean or median, most frequently used measure is standard deviation.
Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list; or the mean of the two central values, if the list contains an even number of values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.*
Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.
Mode. The most frequent value in a set of data.
Model. A mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real- world or mathematical objects, properties, actions, or relationships.
Monomial: An algebraic expression made up of one term.
Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0–100. Example: 72 ÷ 8 = 9.
Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. *Example: 3⁄4 and 4⁄3 are multiplicative inverses of one another because 3⁄4 x 4⁄3 = 4⁄3 x 3⁄4 = 1.*

[2] To be more precise, this defines the arithmetic mean.

N

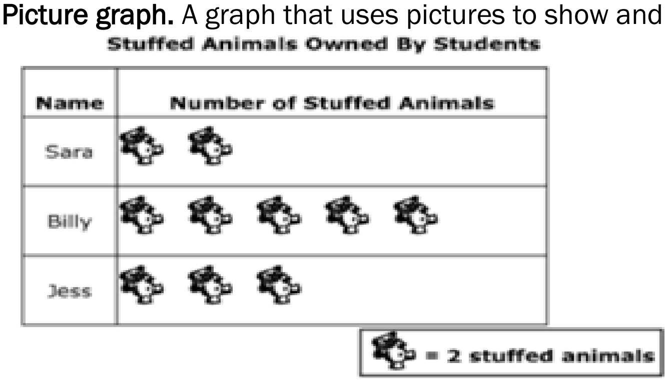
Network. a) A figure consisting of vertices and edges that shows how objects are connected; b) A collection of points (vertices), with certain connections (edges) between them.
Non-linear association. The relationship between two variables is nonlinear if the change in the second is not simply proportional to the change in the first, independent of the value of the first variable.
Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

O

Order of Operations. Convention adopted to perform mathematical operations in a consistent order. 1. Perform all operations inside parentheses, brackets, and/or above and below a fraction bar in the order specified in steps 3 and 4; 2. Find the value of any powers or roots; 3. Multiply and divide from left to right; 4. Add and subtract from left to right.

P

Partition. A process of dividing an object into parts.
Percent rate of change. A rate of change expressed as a percent. *Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.*
Periodic phenomena. Naturally recurring events, for example, ocean tides, machine cycles.
Picture graph. A graph that uses pictures to show and compare information.



Plane. A flat surface that extends infinitely in all directions.
Polar form. The polar coordinates of a complex number on the complex plane. The polar form of a complex number is written in any of the following forms: $r\cos\theta + r\,i\sin\theta$, $r(\cos\theta + i\sin\theta)$, or $r\text{cis}\,\theta$. In any of these forms, r is called the modulus or absolute value. θ is called the argument.
Polynomial: The sum or difference of two or more monomials.
Polynomial function. Any function whose value is the solution of a polynomial.
Postulate. A statement accepted as true without proof.
Prime factorization. A number written as the product of all its prime factors.
Prime number. A whole number greater than 1 whose only factors are 1 and itself.
Probability distribution. The set of possible values of a random variable with a probability assigned to each.
Properties of equality. See Table 4.
Properties of inequality. See Table 5.
Properties of operations. See Table 3.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, testing for a medical condition).
Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also: uniform probability model.*
Proof. A proof of a mathematical statement is a detailed explanation of how that statement follows logically from statements already accepted as true.
Proportion. An equation that states that two ratios are equivalent.
Pythagorean Theorem. For any right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Q

Quadratic equation. An equation that includes only second-degree polynomials. *Some examples are* $y = 3x^2 - 5x^2 + 1$, $x^2 + 5xy + y^2 = 1$, and $1.6a^2 + 5.9a - 3.14 = 0$.
Quadratic expression. An expression that contains the square of the variable, but no higher power of it.
Quadratic function. A function that can be represented by an equation of the form $y = ax^2 + bx + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$. The graph of this function is a parabola.
Quadratic polynomial. A polynomial where the highest degree of any of its terms is 2.

R

Radical. The √ symbol, which is used to indicate square roots or *n*th roots.
Random sampling. A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects.
Random variable. An assignment of a numerical value to each outcome in a sample space.
Ratio. A relationship between quantities such that for every a units of one quantity there are b units of the other. A ratio is often denoted by $a:b$ and read "*a* to *b*".
Rational expression. A quotient of two polynomials with a non-zero denominator.
Rational number. A number expressible in the form a/b or –a/b for some fraction a/b. The rational numbers include the integers. See Illustration 1.
Real number. A number from the set of numbers consisting of all rational and all irrational numbers. See Illustration 1.
Rectangular array. An arrangement of elements into rows and columns.
Rectilinear figure. A polygon all angles of which are right angles.
Recursive pattern or sequence. A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.
Reflection. A type of transformation that flips points about a line, called the line of reflection. Taken together, the image and the pre-image have the line of reflection as a line of symmetry.
Relative frequency. The empirical counterpart of probability. If an event occurs N' times in N trials, its relative frequency is N'/N.
Relatively Prime. Two positive integers that share no common divisors greater than 1; that is, the only common positive factor of the two numbers is 1.
Remainder Theorem. If $f(x)$ is a polynomial in x then the remainder on dividing $f(x)$ by $x - a$ is $f(a)$.
Repeating decimal. A decimal in which, after a certain point, a particular digit or sequence of digits repeats itself indefinitely; the decimal form of a rational number. *See also: terminating decimal.*
Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Rotation. A type of transformation that turns a figure about a fixed point, called the center of rotation.

S

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.
Scatter plot. A graph in the coordinate plane representing a set of bivariate data. *For example, the heights and weights of a group of people could be displayed on a scatter plot.*
Scientific notation. A widely used system where nonzero numbers are written in the form $m \times 10^n$ where n is an integer, and m is a nonzero real number between 1 and 10, e.g., $562 = 5.62 \times 10^2$.
Significant figures (digits). A way of describing how precisely a number is written, particularly when the number is a measurement.
Similarity transformation. A rigid motion followed by a dilation.
Sine (of an acute angle). The trigonometric function that for an acute angle is the ratio between the leg opposite the angle when the angle is considered part of a right triangle and the hypotenuse.
Standard algorithm. A step-by-step approach to calculating, decided by societal convention, developed for efficiency. Flexible and fluent use of standard algorithms requires conceptual understanding.

T

Tangent. a) Meeting a curve or surface in a single point if a sufficiently small interval is considered. b) The trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle and the leg adjacent to the angle when the angle is considered part of a right triangle.
Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.
Terminating decimal. A decimal is called terminating if its repeating digit is 0. A terminating decimal is the decimal form of a rational number. *See also: repeating decimal.*
Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range.*
Transformation. A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the pre-image) and the points in another geometric object (the image). Reflections, rotations, translations, and dilations are particular examples of transformations.
Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.
Translation. A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size.
Trapezoid. A quadrilateral with at least one pair of parallel sides. (Note: There are two definitions for the term *trapezoid*. This is the inclusive definition. For more information see commoncoretools.me/wpcontent/uploads/2014/12/ccss_progression_gk6_2014_12_27.pdf).
Trigonometric function. A function (as the sine, cosine, tangent, cotangent, secant, or cosecant) of an arc or angle most simply expressed in terms of the ratios of pairs of sides of a right-angled triangle.

U

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also: probability model.*

Unit fraction. A fraction with a numerator of 1, such as 1/3 or 1/5.

V

Variable. A quantity that can change or that may take on different values. Refers to the letter or symbol representing such a quantity in an expression, equation, inequality, or matrix.
Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.
Visual fraction model. A tape diagram, number line diagram, or area model.

W

Whole numbers. The numbers 0, 1, 2, 3, . . . See Illustration 1.

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Table 1. Common addition and subtraction situations[1]. This table shows the "situation types" or categories of word problems required by the standards in which the given number and the unknowns are in a variety of configurations. Darker shading [2] indicates the four ***Kindergarten*** problem subtypes. ***Grade 1 and 2*** students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2.

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[3]**
Put Together/ Take Apart[4]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[5]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Note: Additional guidance on addition and subtraction problem types can be found at Quick Reference Guide: Common Addition and Subtraction Situations (mass.edu)

[1]Adapted from Boxes 2–4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32–33).

[2]Adapted from Achievethecore.org :: Situation Types for Operations in Word Problems

[3]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[4]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[5]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2. Common multiplication and division situations[1] (Grades 3 – 5). This table shows the "situation types" or categories of word problems required by the standards in which the given number and the unknowns are in a variety of configurations

	Unknown Product 3 x 6 = ?	Group Size Unknown ("How many in each group?" Division) 3 x ? = 18 and 18 ÷ 3 = ?	Number of Groups Unknown ("How many groups?" Division) ? x 6 = 18 and 18 ÷ 6 = ?
Equal Groups	There are three bags with six plums in each bag. How many plums are there in all? Measurement example: You need three lengths of string, each six inches long. How much string will you need altogether?	If 18 plums are shared equally into three bags, then how many plums will be in each bag? Measurement example: You have 18 inches of string, which you will cut into three equal pieces. How long will each piece of string be?	If eighteen plums are to be packed six to a bag, then how many bags are needed? Measurement example: You have 18 inches of string, which you will cut into pieces that are six inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with six apples in each row. How many apples are there? Area example: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into three equal rows, how many apples will be in each row? Area example: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of six apples, how many rows will there be? Area example: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare[4]	A blue hat costs $6. A red hat costs three times as much as the blue hat. How much does the red hat cost? Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be three times as long?	A red hat costs $18 and that is three times as much as a blue hat costs. How much does a blue hat cost? Measurement example: A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

Note: Additional guidance on multiplication and division problem types can be found at Quick Reference Guide: Common Multiplication and Division Situations (mass.edu)

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in three rows and six columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[4]Multiplicative Compare problems appear first in Grade 4, with whole-number values, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Adapted from Achievethecore.org :: Situation Types for Operations in Word Problems

Table 3. The Properties of Operations Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.	
Associative Property of Addition	(*a* + *b*) + *c* = *a* +(*b* + *c*)
Commutative Property of Addition	*a* + *b* = *b* + *a*
Additive Inverse Property of Zero	*a* + 0 = 0 + *a* = *a*
Existence of Additive Inverses	For every *a* there exists -*a* so that *a* + (-*a*) = (-*a*) + a = 0.
Associative Property of Multiplication	(*a* x *b*) x *c* = *a* x (*b* x *c*)
Commutative Property of Multiplication	*a* x *b* = *b* x *a*
Multiplicative Identity Property of One	*a* x 1 = 1 x *a* = *a*
Existence of Multiplicative Inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* x 1/*a* = 1/*a* x *a* = 1.
Distributive Property of Multiplication over Addition	*a* x (*b* + *c*) = *a* x *b* + *a* x *c*

Table 4. The Properties of Equality Here *a*, *b*, and *c* stand for arbitrary numbers in the rational, real, or complex number systems.	
Reflexive Property of Equality	*a* = *a*
Symmetric Property of Equality	If *a* = *b*, then *b* = *a.*
Transitive Property of Equality	If *a* = *b* and *b* = *c*, then *a* = *c*.
Addition Property of Equality	If *a* = *b*, then *a* + *c* = *b* + *c.*
Subtraction Property of Equality	If *a* = *b*, then *a* - *c* = *b* - *c.*
Multiplication Property of Equality	If *a* = *b*, then *a* x *c* = *b* x *c.*
Division Property of Equality	If *a* = *b* and *c* ≠ 0, then *a* ÷ *c* = *b* ÷ *c.*
Substitution Property of Equality	If *a* = *b*, then *b* may be substituted for *a* in any expression containing *a.*

Table 5. The Properties of Inequality

Here *a*, *b*, and *c* stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: *a* < *b*, *a* = *b*, *a* > *b*.
If *a* > *b* and *b* > *c* then *a* > c.
If *a* > *b*, then *b* < *a.*
If *a* > *b*, then -*a* < -*b.*
If *a* > *b*, then *a* ± *c* > *b* ± *c*.
If *a* > *b* and c > 0, then *a* x *c* > *b* x *c.*
If a > *b* and c < 0, then *a* x *c* < *b* x *c.*
If *a* > *b* and c > 0, then *a* ÷ *c* > *b* ÷ *c.*
If *a* > *b* and c < 0, then *a* ÷ *c* < *b* ÷ *c.*

Table 6. Illustrative Example of an Alternative Algorithm vs. a Standard Algorithm

An **algorithm** is defined as a procedure for solving a mathematical problem in a finite number of steps that frequently involves repetition of an operation.

The standards require mastery of several U.S. Standard Algorithms for efficiency at specified grade levels. However, the standards are constructed so students' understanding and ability to use standard algorithms is predicated on the development of their conceptual understanding through experiences exploring and using a variety of alternative algorithms.

This table provides **one** example of an alternative algorithm for the operation of addition. High-quality curriculum materials (HQCM) will offer other alternatives when teaching addition. Furthermore, HQCM will include alternative algorithms for the operations of subtraction, multiplication, and division. The use of alternative algorithms aids in developing students' conceptual understanding of an operation.

One Example of an Alternative Algorithm for Addition*	Standard Algorithm for Addition (for efficiency)
356 +167 400 (Sum of hundreds) 110 (Sum of tens) 13 (Sum of ones) 523	11 (Regrouped 10 and 100) 356 +167 523

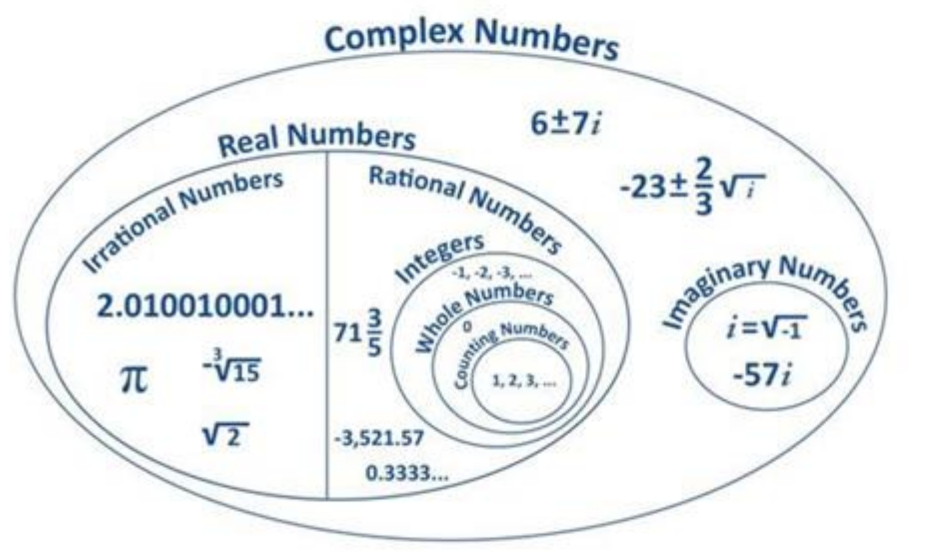
Note: Additional guidance on algorithms for addition and subtraction can be found at [Quick Reference Guide (QRG) - Algorithms Sept 2017 (mass.edu)]
Additional guidance for algorithms for multiplication and division can be found at [Quick Reference Guide: Standard Algorithms for Multiplication and Division (mass.edu)]

Table 7. Required Grade-Level Fluencies (Grades K – 6)
This table itemizes the fluencies required by the standards for grades K through 6. Fluency is **defined** as using efficient, flexible and accurate methods of computing. Table is adapted from Achievethecore.org :: Instructional Content Nav - Mathematics: Focus by Grade Level

Grade	Standard	Required Fluency
K	K.OA.A.5	Add/subtract within 5
1	1.OA.C.6	Add/subtract within 10
2	2.OA.B.2 2.NBT.B.5	Single-digit sums and differences (sums by memory by end of grade) Add/subtract within 100
3	3.OA.C.7 3.NBT.A.2	Single-digit products and quotients (products by memory by end of grade) Add/subtract within 1000
4	4.NBT.B.4	Add/subtract within 1,000,000
5	5.NBT.B.5	Multi-digit multiplication
6	6.NS.B.2 6.NS.B.3	Multi-digit division Multi-digit decimal operations

Illustration 1. The Number System*

The Number System is comprised of number sets beginning with the Counting Numbers and culminating in the more complete Complex Numbers. The name of each set is written on the boundary of the set, indicating that each increasing oval encompasses the sets contained within. Note that the Real Number Set is comprised of two parts: Rational Numbers and Irrational Numbers.



*Adopted from the Massachusetts Model Content Frameworks for Mathematics, 2017.