Mathematics

CURRICULUM FRAMEWORK

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RIDE Rhode Island Department of Education

Table of Contents

Section 1: Introduction

Background

The Rhode Island Department of Education (RIDE) is committed to ensuring all students have access to high-quality curriculum and instruction as essential components of a rigorous education that prepares every student for success in college and/or their career. Rhode Island's latest strategic plan outlines a set of priorities designed to achieve its mission and vision. Among these priorities is *Excellence in Learning*. In 2019, **Rhode Island General Law (RIGL) § 16-22-31** was passed by the state legislature, as part of [Title 16 Chapter 97 - The Rhode Island Board of Education Act], signaling the importance of *Excellence in Learning* via high-quality curriculum and instruction. RIGL § 16-22-31 requires the Commissioner of Elementary and Secondary Education and RIDE to develop statewide curriculum frameworks that support high-quality teaching and learning.

The mathematics curriculum framework is specifically designed to address the criteria outlined in the legislation, which includes, but is not limited to, the following: providing sufficient detail to inform education processes such as selecting curriculum resources and designing assessments; encouraging real-world applications; being designed to avoid the perpetuation of gender, cultural, ethnic, or racial stereotypes; and presenting specific, pedagogical approaches and strategies to meet the academic and nonacademic needs of multilingual learners.[1]

The mathematics framework was developed by an interdisciplinary team through an open and consultative process. This process incorporated feedback from a racially and ethnically diverse group of stakeholders that included members of the RIDE Mathematics Advisory Board, students, families, the general public, and community partners.

Vision for Student Success in Mathematics

RIDE envisions an educational landscape in which each Rhode Island graduate is well prepared to lead a fulfilling and productive life, succeed in academic and employment settings, and contribute meaningfully to society. He or she can think critically and collaboratively and act as a creative, self-motivated, culturally and globally competent learner (RIDE Strategic Plan, 2021).

Proficiency in mathematics is one of the key characteristics of a well-prepared graduate. As such, the goal of the mathematics framework is to create conditions in which all students will gain the ability to reason, communicate, and effectively problem-solve with mathematics. Through the use of research-based strategies, RIDE will establish policy and guidance to build our students' appreciation for, understanding of, and ability to apply mathematics in the real world, thus preparing them to be successful and actively engaged citizens in the real world.

Purpose

The purpose of the mathematics framework is to provide guidance to educators and families around the implementation of the standards, particularly as it relates to the design and use of curriculum materials, instruction, and assessment. The frameworks should streamline a vertical application of standards and assessment across the K–12 continuum within Tier 1 of a Multi-Tier System of Support (MTSS), increase opportunities for all students to meaningfully engage in grade-level work and tasks, and ultimately support educators and families in making decisions that prioritize the student experience. These uses of the curriculum frameworks align with the overarching

[1] The legislation uses the term *English learners*; however, RIDE has adopted the term *multilingual learners* (MLLs) to refer to the same group of students to reflect the agency's assets-based lens.

commitment to ensuring all students have access to high-quality curriculum and instruction that prepares students to meet their postsecondary goals.

Success Criteria

This framework should support educators in accomplishing the following:

- ✓ Equitably and effectively support the learning of all students, including multilingual learners and differently-abled students.
- ✓ Support and reinforce the importance of culturally responsive and sustaining education practices.
- ✓ Prepare students to thrive and succeed in college and/or their careers.

Guiding Principles for Rhode Island's Frameworks

The following five guiding principles are the foundation for Rhode Island's Curriculum Frameworks. They are intended to frame the guidance within this document around the use and implementation of standards to drive curriculum, instruction, and assessment within an MTSS. These principles include the following:

1. Standards are the bedrock of an interrelated system involving high-quality curriculum, instruction, and assessment.
2. High-quality curriculum materials (HQCMs) align to the standards and, in doing so, must be accessible, culturally responsive and sustaining, supportive of multilingual learners, developmentally appropriate, and equitable, as well as leverage students' strengths as assets.
3. High-quality instruction provides equitable opportunities for all students to learn and reach proficiency with the knowledge and skills in grade-level standards by using engaging, data-driven, and evidence-based approaches such as leveraging home languages for content learning and drawing on family and communities as resources.
4. To be valid and reliable, assessments must align to the standards and equitably provide students with opportunities to monitor learning and demonstrate proficiency.
5. All aspects of a standards-based educational system, including policies, practices, and resources, must work together to support all students, including multilingual learners and differently-abled students.

What is 'Curriculum'?

A common misconception about school curricula is the belief that a curriculum is primarily the collection of resources used to teach a specific course or subject. A high-quality curriculum is much more than this. RIDE has previously defined curriculum as a "standards-based sequence of planned experiences where students practice and achieve proficiency in content and applied learning skills. Curriculum is the central guide for all educators as to what is essential for teaching and learning, so that every student has access to rigorous academic experiences." Building off this definition, RIDE also identifies specific components that comprise a complete curriculum. These include the following:

- ***Goals***: Goals within a curriculum are the standards-based benchmarks or expectations for teaching and learning. Most often, goals are made explicit in the form of a scope and sequence of skills to be addressed. Goals must include the breadth and depth of what a student is expected to learn.
- ***Instructional Practices***: Instructional practices are the research and evidence-based methods (i.e., decisions, approaches, procedures, and routines) that teachers use to engage all students in meaningful learning. These choices support the facilitation of learning experiences in order to promote a student's ability to understand and apply content and skills. Strategies are differentiated to meet student needs and interests, task demands, and learning environment. They are also adjusted based on ongoing review of student progress towards meeting the goals.
- ***Materials***: Materials are the tools and resources selected to implement methods and achieve the goals of the curriculum. They are intentionally chosen to support a student's learning, and the selection of resources should reflect student interest, cultural diversity, world perspectives, and address all types of diverse learners. To assist local education agencies (LEAs) with the selection process, RIDE has identified and approved a collection of HQCMs in mathematics and English language arts (ELA) in advance of the 2023 selection and adoption requirement for LEAs. The intent of this list is to provide LEAs with the ability to choose a high-quality curriculum that best fits the needs of its students, teachers, and community. Each LEA must choose a curriculum from the list for core mathematics, ELA, and science content areas per the timelines outlined in RIGL§ 16.22.30-33. When possible, LEAs should adopt early because every student in Rhode Island deserves access to HQCMs.
- ***Assessment***: Assessment in a curriculum is the ongoing process of gathering information about a student's learning. This includes a variety of ways to document what the student knows, understands, and can do with their knowledge and skills. Information from assessment is used to make decisions about instructional approaches, teaching materials, and academic supports needed to enhance opportunities for the student and to guide future instruction.

Another way to think about curriculum, and one supported by many experts, is that a well-established curriculum consists of three interconnected parts all tightly aligned to standards: the intended (or written) curriculum, the lived curriculum, and the learned curriculum (e.g., Kurz, Elliott, Wehby, & Smithson, 2010). Additionally, a cohesive curriculum should ensure that teaching and learning is equitable, culturally responsive and sustaining, and offers students multiple means through which to learn and demonstrate proficiency.

The *written curriculum* refers to what students are expected to learn as defined by standards, as well as the HQCMs used to support instruction and assessment. This aligns with the 'goals' and 'materials' components described above. Given this, programs and textbooks do not comprise a curriculum on their own, but rather are the resources that help to implement it. They also establish the foundation of students' learning experiences. The written curriculum should provide students with opportunities to engage in content that builds on their background experiences and cultural and linguistic identities while also exposing students to new experiences and cultural identities outside of their own.

The *lived curriculum* refers to how the written curriculum is delivered and assessed and includes *how* students experience it. In other words, the lived curriculum is defined by the quality of instructional practices that are applied when implementing the HQCMs. This aligns with the 'methods' section in RIDE's curriculum definition. The lived curriculum must promote instructional engagement by affirming and validating students' home culture and language, as well as provide opportunities for integrative and interdisciplinary learning. Content and tasks should be instructed through an equity lens, providing educators and students with the opportunity to confront complex equity issues and explore socio-political identities.

Finally, the *learned curriculum* refers to how much of and how well the intended curriculum is learned and how fully students meet the learning goals as defined by the standards. This is often defined by the validity and reliability of assessments, as well as by student achievement, their work, and performance on tasks. The learned curriculum should reflect a commitment to the expectation that all students can access and attain grade-level proficiency. Ultimately, the learned curriculum is an expression and extension of the written and lived curricula, and should promote critical consciousness in both educators and students, providing opportunities for educators and students to improve systems for teaching and learning in the school community.

Key Takeaways

- First, the **written curriculum** (goals and HQCMs) must be firmly grounded in the standards and include a robust set of HQCMs that all teachers know how to use to design and implement instruction and assessment for students.
- Second, the characteristics of a strong **lived curriculum** include consistent instructional practices and implementation strategies that take place across classrooms that are driven by standards, evidence-based practices, learning tasks for students that are rigorous and engaging, and a valid and reliable system of assessment.
- Finally, student learning and achievement are what ultimately define the overall strength of a **learned curriculum**, including how effectively students are able to meet the standards.

What is a Curriculum Framework?

All of Rhode Island's curriculum frameworks are designed to provide consistent guidance around how to use standards to support the selection and use of HQCMs, evidence-based instructional practices, as well as valid and reliable assessments — all in an integrated effort to equitably maximize learning for all students.

The curriculum frameworks include information about research-based, culturally responsive and sustaining, and equitable pedagogical approaches and strategies for use during implementation of HQCMs and assessments in order to scaffold, develop, and assess the skills, competencies, and knowledge called for by the state standards.

The structure of this framework also aligns with the five guiding principles referenced earlier. **Section 2** lists the standards and provides a range of resources to help educators understand and apply them. Section 2 also addresses how standards support selection and implementation of HQCMs. **Section 3** of this framework provides guidance and support around how to use the standards to support high-quality instruction. **Section 4** offers resources and support for using the standards to support assessment. Though Guiding Principle 5 does not have a dedicated section, it permeates the framework. Principle 5 speaks to the coherence of an educational system grounded in rigorous standards. As such, attention has been given in this framework to integrate stances and resources that are evidence-based, specific to the standards, support the needs of all learners — including multilingual learners and differently-abled students — and link to complementary RIDE policy, guidance, and initiatives. Principle 5 provides the vision of a coherent, high-quality educational system.

In sum, each curriculum framework, in partnership with HQCMs, informs decisions at the classroom, school, and district level about curriculum material use, instruction, and assessment in line with current standards and with a focus on facilitating equitable and culturally responsive and sustaining learning opportunities for all students. The curriculum frameworks can also be used to inform decisions about appropriate foci for professional learning, certification, and evaluation of active and aspiring teachers and administrators.

The primary audiences for the information and resources in the curriculum frameworks are educators in Rhode Island who make decisions and implement practices that impact students' opportunities for learning in line with standards. This means that the primary audience includes teachers, instructional leaders, and school and district administrators.

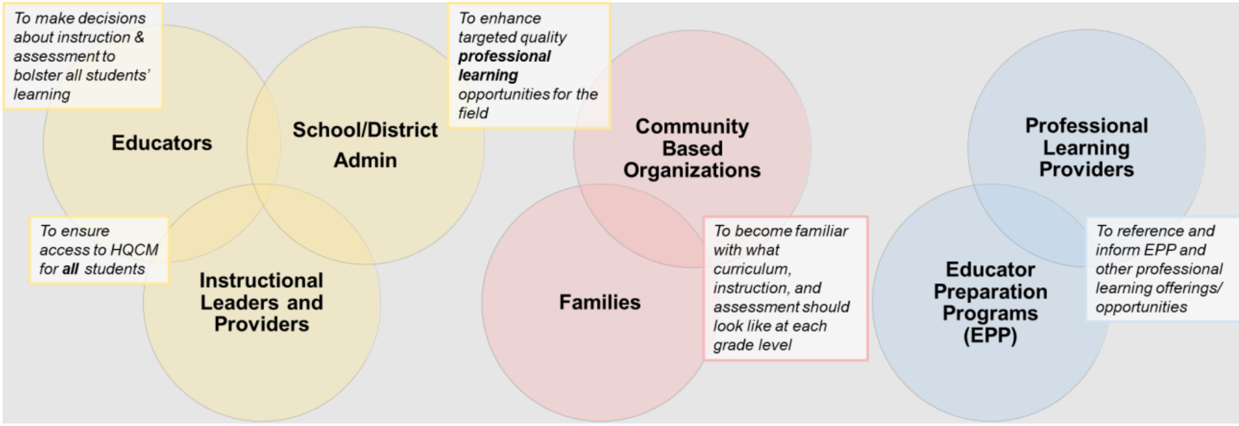
However, the curriculum frameworks also provide an overview for the general public, including families and community members, about what equitable standards-aligned curriculum, instruction, and assessment should look like for students in Rhode Island. They also serve as a useful reference for professional learning providers and higher education Educator Preparation Programs (EPPs) offering support for Rhode Island educators. Thus, this framework is also written to be easily accessed and understood by families and community members.

Summary of Section Structure

Section II: Implementing a High-Quality Curriculum	Section III: Implementing High-Quality Instruction	Section IV: High-Quality Learning Through Assessment
✓ The Practice Standards ✓ Content-Area Standards ✓ Connecting the Content and Practice Standards* ✓ Suggested Pathways/Progressions ✓ Professional Learning ✓ Selecting HQ Curriculum Materials*	✓ Principles of HQ Instruction ✓ HQ Cross-Content Instructional Practices ✓ Implementing HQ Lessons* ✓ Work-Based Learning* ✓ Professional Learning	✓ Principles of HQ Assessment ✓ Designing HQ Performance Assessments ✓ Designing HQ Rubrics ✓ Assessing Work-Based Learning ✓ Professional Learning

What does effective implementation of the Curriculum Framework look like?

Below are examples of how RIDE envisions the guidance and resources within this framework being used. These examples are not exhaustive by any measure and are intended to give educators an initial understanding of how to practically begin thinking about how to implement and use this framework to inform their daily practice.



Educators and instructional leaders such as curriculum coordinators, principals, and instructional coaches can use the curriculum frameworks as a go-to resource for understanding the HQCMs that have been adopted in their districts and to make decisions about instruction and assessment that bolster all students' learning opportunities. For example, the frameworks can be used to:

- Unpack and internalize grade-level standards and vertical alignment of the standards;
- Analyze HQCMs and assessment(s) adopted in the district and understand how the standards are applied within the instructional materials and assessment(s);
- Norm on high-quality instructional practices in each of the disciplines; and
- Guide decisions related to instruction and assessment given the grade-level expectations for students articulated in the standards and the high-quality instructional materials.

Educators, curriculum leaders, and instructional coaches can use the curriculum frameworks as a resource when ensuring access to high-quality instructional materials for all students that are culturally responsive and sustaining, and that equitably and effectively include supports for MLLs such as home language materials when available. For example, the frameworks can be used to:

- Unpack and internalize English language development standards for MLLs; and
- Plan universally designed instruction and aligned scaffolds that ensure all students can engage meaningfully with grade-level instruction.

District and school administrators can use the curriculum frameworks to calibrate their understanding of what high-quality curriculum, instruction, and assessment should look like within and across disciplines and use that understanding as a guide to:

- Make resources available to educators, families, and other stakeholders in support of student learning;
- Norm "what to look for" in classrooms as evidence that students are receiving a rigorous and engaging instructional experience; and
- Structure conversations with teachers and families about high-quality curriculum, instruction, and assessment.

District and school administrators, as well as EPPs and professional learning providers, can use the curriculum frameworks to enhance targeted quality professional learning opportunities for the field. For example, the frameworks can be used to:

- Enhance educator or aspiring educator knowledge about the standards and pedagogical approaches used in Rhode Island;
- Roll out a vision for curriculum and instruction in the district, followed by curriculum-specific professional learning;
- Build capacity of educators and aspiring educators to engage in meaningful intellectual preparation to support facilitation of strong lessons;
- Aid educators and aspiring educators in making sense of the structure, organization, and pedagogical approaches used in different curriculum materials; and
- Build capacity of educators and aspiring educators to address individual learning needs of students through curriculum-aligned scaffolds.

Families and community organizations can use the curriculum frameworks to become familiar with what curriculum, instruction, and assessment should look like at each grade level.

Overview and Connection to Other Frameworks

Each content area (mathematics, science and technology, English language arts/literacy, history and social studies, world languages, and the arts) has, or will soon, have its own curriculum framework. For educators who focus on one content area, all information and resources for that content area are contained in its single curriculum framework. For educators and families who are thinking about more than one content area, the different content-area curriculum frameworks will need to be referenced. However, it is important to note that coherence across the curriculum frameworks includes a common grounding in principles focused on connections to content standards and providing equitable and culturally responsive and sustaining learning opportunities through curriculum resources, instruction, and assessment. The curriculum frameworks also explicitly connect to RIDE's work in other areas including, but not limited to, MLLs, differently-abled students, early learning, college and career readiness, and culturally responsive and sustaining practices. Below is a brief overview of how this and the other curriculum frameworks are organized, as well as a summary of how the specific curriculum frameworks overlap and connect to each other.

Section	What is common across the content area curriculum frameworks?	What is content-specific in each content area's curriculum framework?
Section 1: Introduction	Section 1 provides an overview of the context, purpose, and expectations related to the curriculum framework.	Each curriculum framework articulates a unique **vision for how the framework can support high-quality teaching and learning.**
Section 2: Implementing a High-Quality Curriculum	The introduction to this section defines how RIDE defines HQCMs in relation to standards.	The middle section of each curriculum framework has **content-specific information about the standards behind curriculum resources and the vision for student**

Section	What is common across the content area curriculum frameworks?	What is content-specific in each content area's curriculum framework?
	The final part of this section explains how HQCMs are selected in RI and provides related tools.	**success** in the targeted content area. The final part of this section includes some **specific information about the HQCMs for the targeted content area.**
Section 3: Implementing High-Quality Instruction	This section provides an overview of how high-quality instruction is guided by standards and introduces five cross-content instructional practices for high-quality instruction. This section also includes guidance and tools to support high-quality instruction and professional learning across content areas.	This section expands upon the cross-content instructional practices by providing **content-specific information about instructional practices.** This section also includes **more specific guidance and tools for considering instruction and professional learning in the targeted content area.**
Section 4: High-Quality Learning Through Assessment	The curriculum frameworks are all grounded in common information described here about the role of formative and summative assessment and how these align with standards. Some standard tools and guidance for assessment in any content area are also provided.	**Content-specific guidance about tools and resources for assessing students in the targeted content area** are included in this section.

Connections to Other RIDE Resources

This curriculum framework is designed to be a valuable resource for educators and families. It is intended to support classroom teachers and school leaders in developing a robust and effective system of teaching and learning. To achieve this, it also connects users to the vast array of guidance and resources that the RIDE has and will continue to develop. Thus, when logical, direct references are made, including direct hyperlinks, to any additional resources that will help educators, families, and community members implement this framework. Of particular significance is the link to college and career readiness.

College and Career Readiness

RIDE's mission for College and Career Readiness is to build an education system in Rhode Island that prepares all students for success in college and career. This means that all doors remain open and students are prepared for whatever their next steps may be after high school.

Secondary education, which begins in middle school and extends through high school graduation, is the point in the educational continuum where students experience greater choice on their journey to college and career readiness. Students have access to a wide range of high-quality personalized learning opportunities and academic coursework, and have a variety of options available to complete their graduation requirements. To improve student engagement and increase the relevance of academic content, students may choose to pursue a number of courses and learning experiences that align to a particular area of interest, including through dedicated career and technical education programs or early college coursework opportunities.

Secondary level students have opportunities to be able to control the pace, place, and content of their learning experience while meeting state and local requirements. Rhode Island middle and high school students will have access to a wide range of high-quality early college and early career training programs that enable them to earn high-value, portable credit and credentials.

References

Kurz, A., Elliott, S. N., Wehby, J. H., & Smithson, J. L. (2010). Alignment of the Intended, Planned, and Enacted Curriculum in General and Special Education and Its Relation to Student Achievement. *The Journal of Special Education*, *44*(3), 131-145. Retrieved from Alignment-of-the-Intended-Planned-and-Enacted-Curriculum-in-General-and-Special-Education-and-Its-Relation-to-Student-Achievement.pdf (researchgate.net)

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Section 2: Implementing a High-Quality Curriculum

Introduction

Having access to high-quality curriculum materials (HQCMs) is an important component of increasing equitable access to a rigorous education that prepares every student for college and careers. In answer to this national movement to increase access through high-quality materials, the State of Rhode Island, in 2019, passed RIGL§ 16.22.30-33. The legislation requires that all Rhode Island Local Education Agencies (LEAs) adopt HQCMs in K–12 schools that are (1) aligned with academic standards, (2) aligned with the curriculum frameworks, and (3) aligned with the statewide standardized test(s), where applicable.

RIDE uses various factors to determine high quality, primarily using information from EdReports, a non-profit, independent organization that uses teams of trained teachers to conduct reviews of K–12 English language arts (ELA), mathematics, and science curricula. Informed by EdReports as a baseline, RIDE's list includes only curricula that are rated "Green" in all three gateways: (1 & 2) alignment to standards with depth and quality in the content area, and (3) usability of instructional materials for teachers and students. Because EdReports' gateways comprise many indicators, which provide more in-depth looks across the integral components of instructional materials, it is important to note that having a "Green-rated" curriculum is a solid foundation, yet not enough on its own to ensure alignment to local instructional priorities and students' needs. The curriculum adoption process should include consideration of an LEA's instructional vision, multilingual learner (MLL) needs, and culturally responsive and sustaining education (CRSE). Selection is only the starting point in the larger process of adoption and implementation of high-quality instructional materials. LEAs should consider curriculum adoption and implementation an iterative process where the efficacy of a curriculum is reviewed and evaluated on an ongoing basis.

Coherence is one major consideration when adopting a new curriculum. One way of achieving coherence is the vertical articulation in a set of materials, or the transition and connection of skills, content, and pedagogy from grade to grade. Consideration of coherence is necessary to ensure that students experience a learning progression of skills and content that build over time through elementary, middle, and high school. As such, LEAs who consider the adoption of curriculum materials are cautioned against choosing a curriculum that is high quality at only one grade level, as it is likely it will disrupt a cohesive experience in the learning progression from grade to grade in the school or district.

While the standards describe what students should know and be able to do, they do not dictate how they should be taught, or the materials that should be used to teach and assess those (NGA & CCSSO, 2010). Curriculum materials, when aligned to the standards, provide students with varied opportunities to gain the knowledge and skills outlined by the standards. Assessments, when aligned to the standards, have the goal of understanding how student learning is progressing toward acquiring proficiency in the knowledge and skills outlined by the standards as delivered by the curriculum through instruction (CSAI, 2018).

No set of grade-level standards can reflect the great variety of abilities, needs, learning rates, and achievement levels in any given classroom. The standards define neither the support materials that some students may need nor the advanced materials that others should have access to. It is also beyond the scope of the standards to define the full range of support appropriate for MLLs and for differently-abled students. Still, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills that will be necessary in their postsecondary lives. The standards should be read as allowing for the widest possible range of students to participate fully from the outset with appropriate accommodations to ensure maximum participation of students, particularly those from historically underserved populations (MDOE, 2017).

Having access to HQCMs is an important component of increasing equitable access to a rigorous education that prepares every student for college and careers.

The Rhode Island Core Standards for Mathematics

The Rhode Island Core Standards for Mathematics (RICSM) are organized into two separate but interrelated bodies of standards: Standards for Mathematical Practice and Standards for Mathematical Content. The content standards are subdivided by grade level for grades K–8, and by conceptual category for high school.

The RICSM were endorsed as the state standards by the Council for Elementary and Secondary Education on March 9, 2021. Grounded in both the Common Core State Standards for Mathematics and the Massachusetts Curriculum Framework for Mathematics (2017), the standards are research-based and informed by instructional practice (MDOE, 2017).

The RICSM define what students should understand and be able to do in their study of mathematics at specific grade levels or grade spans. They identify the skills, knowledge, and competencies students should have to succeed in entry-level, credit-bearing academic college courses and in workforce training programs and provide the foundation for high-quality curriculum (NGA & CCSSO, 2010). While the standards are divided into content and practice standards, it is understood that these two sets of standards must always be connected to promote a deep understanding of and the ability to do and apply mathematics.

Essential Attributes of the Rhode Island Core Standards for Mathematics: Focus, Coherence, and Rigor

The RICSM build on the best of existing standards and reflect the skills and knowledge students will need to succeed in college, career, and life. These standards embody three essential attributes: focus, coherence, and rigor. Understanding the essential attributes is pivotal to implementing them.

Greater Focus on Fewer Topics

The RICSM call for greater focus in mathematics. Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask mathematics teachers to significantly narrow and deepen the way time and energy are spent in the classroom. This means focusing deeply on the major work of each grade.

This focus will help students gain strong foundations, including a solid understanding of concepts, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems inside and outside the classroom.

Coherence: Linking Topics and Thinking across Grades

Mathematics is not a list of disconnected topics, tricks, or mnemonics; it is a coherent body of knowledge made up of interconnected concepts. Therefore, the standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. For example, in 4th grade, students must "apply and extend previous understandings of multiplication to multiply a fraction by a whole number" (Standard 4.NF.B.4). This extends to 5th grade, when students are expected to build on that skill to "apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction" (Standard 5.NF.B.4). Each standard is not a new event, but an extension of previous learning.

Coherence is also built into the standards in how they reinforce a major topic in a grade by utilizing supporting, complementary topics. For example, instead of presenting the topic of data displays as an end in and of itself, the topic is used to support grade-level word problems in which students apply mathematical skills to solve problems.

***Rigor*: Pursue Conceptual Understanding, Procedural Skills and Fluency, and Application with Equal Intensity**

Rigor refers to deep, authentic command of mathematical concepts, not making mathematics harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

- *Conceptual understanding:* The standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives in order to see mathematics as more than a set of mnemonics or discrete procedures.
- *Procedural skills and fluency:* The standards call for speed and accuracy in calculation. Students must practice core functions, such as single-digit multiplication, in order to have access to more complex concepts and procedures. Fluency must be addressed in the classroom or through supporting materials, as some students might require more practice than others.
- *Application:* The standards call for students to use mathematics in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency (Common Core State Standards Initiative, 2020).

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students (NGA and CCSSO, 2010). They describe ways in which developing student practitioners of the discipline of mathematics engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years (MDOE, 2017).

The Standards for Mathematical Practice rest on important "processes and proficiencies" with long-standing importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy) (NGA and CCSSO, 2010).

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* — to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents — and the ability to *contextualize* — to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Standard 3.NF.3a is one such example where this opportunity is presented: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Resource	Description
Mathematics: Focus by Grade Level	Designed by Achieve the Core, these handy K–8 grade-level guides identify the major content of a grade and the required grade-level fluencies.
Quick Reference Guides for Mathematical Practice Quick Reference Guide: Standards for Mathematical Practice Pre-K-2 (mass.edu) Quick Reference Guide: Standards for Mathematical Practice Grades 3-5 (mass.edu) Quick Reference Guide: Standards for Mathematical Practice Grades 6-8 (mass.edu) Quick Reference Guide: Standards for Mathematical Practice High School (mass.edu)	Developed by the Massachusetts Department of Education (MDOE), these guides describe how mathematically proficient students in designated grade bands might demonstrate the eight Standards for Mathematical Practice.

Resource	Description
Standards for Mathematical Practices Progression through Grade Levels (weebly.com)	
Implementing the Mathematical Practice Standards	Created by EDC with funding from the National Science Foundation, this website provides a wealth of information to increase educators' understanding of the practice standards specific to grades 5 through high school. Videos, tasks, and samples of student dialogue are featured.
Implementing Standards for Mathematical Practices	This guide created by Achieve the Core offers a series of questions teachers can use to develop student expertise with the practice standards. It also describes what tasks linked to each practice standard might look like for students, as well as accompanying teacher actions.
Quick Reference Guide: Fractions Learning Progression in Grades 3–5	Created by MDOE, this guide summarizes the progression of standards related to fractions for students in grades 3–5. Accompanying examples of fraction models illustrate how educators can help students deepen conceptual understanding of fractions.

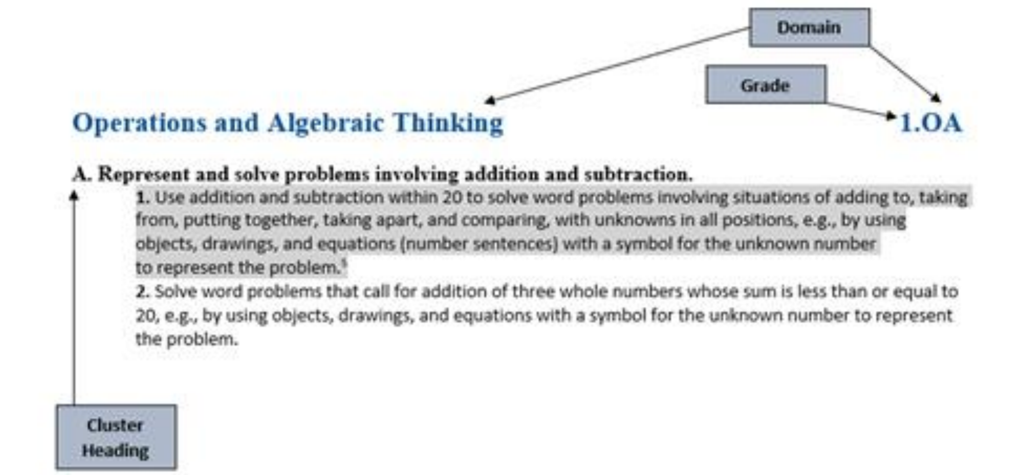
Standards for Mathematical Content

How to Read the Content Standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.



The first highlighted standard above is identified as 1.OA.A.1, identifying it as a grade 1 standard in the Operations and Algebraic Thinking domain, and as the first standard in that domain. Standard 1.OA.A.1 is the first standard in this cluster of standards. All of the RICSM for grades K–8 use the same coding system. Coding for the high school standards differs slightly in that the grade designation is replaced with a conceptual category designation.

These standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade does not necessarily mean that topic A must be taught before topic B. A curriculum might be organized such that topic B is taught before topic A, or the connections between the two topics might be highlighted by teaching each topic simultaneously.

High School Model Mathematics Courses for Algebra 1, Geometry, & Algebra 2 (AGA)

In high school, the Rhode Island Core Standards for Mathematics (RICSM) are organized by conceptual category rather than by grade level. RIDE analyzed a variety of high-quality curricula and consulted with leading mathematics educators in Rhode Island to design model high school courses for Algebra 1, Geometry, and Algebra 2. The model courses outline the general content expectations for the traditional AGA sequence. The model courses and guidance in utilizing them can be found in in the high school section of the RICSM.

WIDA ELD Standards for MLLs

For educators with one or more active MLLs on their roster, enacting standards-aligned instruction means working with both state-adopted content standards and state-adopted English language development (ELD) standards. Under ESSA, all educators are required to reflect on the language demands of their grade-level content and move MLLs toward both English language proficiency and academic content proficiency. In other words, every Rhode Island educator shares responsibility for promoting disciplinary language development through content instruction.

Fortunately, the five WIDA ELD Standards lend themselves to integration in the four core content areas. Standard 1 is cross-cutting and applicable in every school context, whereas Standards 2–5 focus on language use in each of the content areas. Standard 3 is dedicated to the language for mathematics. Educators of mathematics are thus expected to support Standard 1 and Standard 3 as part of their core classroom instruction.

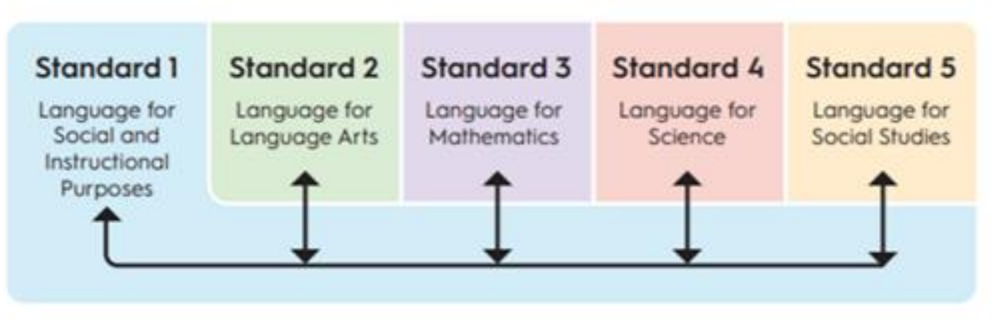


Image Source: [2020 Edition of WIDA ELD Standards Framework]()

Each of the WIDA ELD Standards is broken into four genre families: *Narrate*, *Inform*, *Explain*, and *Argue*. WIDA refers to these genre families as **Key Language Uses** (KLUs) and generated them based on an analysis of the language demands placed on students by the academic content standards. The KLUs are important because they drive explicit language instruction in each of the content areas. For Standards 2–5, the distribution of KLUs is similar across grades 4–12, but this distribution varies in the early grades, with grades K–3 placing more emphasis on *Inform* than *Explain* and *Argue*. Though present, *Narrate* is not a prominent KLU in mathematics.

Each KLU is further broken down by language function and feature. **Language functions** reflect the dominant practices for engaging in genre-specific tasks (e.g., students often make conjectures about patterns or rules in mathematics by relating different cases to one another). By contrast, the **language features** represent a sampling of linguistic and non-linguistic resources (e.g., connected clauses, noun phrases, tables, graphs) that students might use when performing a particular language function. Together, the KLUs, language functions, and language features capture what it would look and sound like for students to use language deftly in mathematics. Please see below for an example of how these three elements appear in the WIDA ELD Standards.

GRADES
2-3 WIDA ELD STANDARD 3
Language for Mathematics

Argue

Language Expectations: Multilingual learners will...

ELD-MA.2-3.Argue.Interpretive	**ELD-MA.2-3.Argue.Expressive**
Interpret mathematics arguments by • Identifying conjectures about what might be true • Distinguishing connections among ideas in justifications • Extracting mathematical operations and facts from solution strategies to create generalizations	Construct mathematics arguments that • Create conjecture using definitions • Generalize commonalities across cases • Justify conclusion steps and strategies in simple patterns • Identify and respond to others' arguments

Language Functions and Sample Language Features

Create conjecture using definitions through...

- Relating verbs *(have, belong to, be)* to make claim *(A is bigger than B because it is taller.)*
- Adverbial phrases (qualities, quantities, frequency) to add precision related to conjecture *(All squares have 4 equal sides. Triangles always have 3 sides.)*

Generalize by finding commonalities across cases through...

- A variety of structures such as comparatives *(er, est; more, most)*; demonstratives *(these, both, that)* to point out similarities *(Both squares and rhombuses have 4 equal sides, ½ is bigger than ¼)*
- Conditional structures *(if/then, when)* to draw conclusions *(If 34+68=102 then 102-68=34, When a number is even you can divide it into two equal parts.)*

Justify conclusion steps and strategies in simple patterns through...

- Technical nouns to add precision and details *(place value, communicative property, angles, measurement, fractions, even/odd)*
- Causal connectors *(because, so, that means)* to present case to others *(The taller rectangle isn't always bigger because you have to look at the area inside.)*
- Drawings, manipulatives, models, diagrams to support thinking

Identify and respond to others' arguments through...

- Questions *(how, what, why)* to ask for clarification or information *(How did you get your answer?)*
- Declarative statements to disagree/debate *(I disagree, I'm not sure, I got a different answer...)*
- Declarative statements to counter claim or reasoning *(5-3 is not the same as 3-5, Just because it has 4 sides that doesn't make it a square, the sides have to be equal)*

Image Source: [2020 Edition of WIDA ELD Standards Framework]

The 2020 Edition of the WIDA ELD Standards Framework contains other resources, such as annotated language samples, that can support educators in promoting integrated language development in mathematics. The annotated language samples show the language functions and language features in action with grade-level texts, as shown in the example below for the KLU

Explain in grades 9–12 mathematics. It offers insights into how educators might unpack the language of their discipline for the KLU *Explain* in grades 9–12 mathematics.

GRADES **9-12** **WIDA ELD STANDARD 3** Language for Mathematics **Explain**

Annotated Language Sample

Context: This mentor explanation text was developed by a teacher to illustrate the language required for students to answer this type of Algebra 1 problem they might encounter on a final exam or standardized test. The teacher read, deconstructed, and analyzed the mentor text with students to make visible how the text is structured as well as the way certain language features are employed to meet the purpose of the explanation.

Prompt: Give the domain and range of the relationship. Then tell whether the relation is a function. Explain your answer. $Y = X^2 - 5$

Language Expectation: ELD-MA.9-12.Explain.Expressive
Multilingual learners use language to construct mathematical explanations that

- Introduce mathematical concept or entity
- Share solutions with others
- Describe data and/or approach used to solve a problem
- State reasoning used to generate own or alternate solutions

Functions & Features	Grade 10 Algebra	Functions & Features
Introduce a concept through... Relating verbs to define or describe concept • is, is, is, Mathematical terms and phrases to describe concept, process • the equation's domain and range • the vertical line test • X • Y	I explain that this equation is a function by providing a definition of the equation's domain and range and graphing what it looks like. This helps me see if the equation passes the vertical line test. The domain is the set of all numbers that can be substituted for X in the equation. X can be any real number, so the domain is all real numbers. The range is all the possible numbers that can be Y.	**Share solutions with others through..** Generalized nouns to add precision • equation • function • domain • range First person to describe approach • I • me Third person to describe approach with neutral stance of authority • the domain • the range

Image Source: [2020 Edition of WIDA ELD Standards Framework]

Selecting High-Quality Curriculum Materials

Selecting and Implementing a High-quality Curricula In Rhode Island: A Guidance Document: This guidance document outlines the provisions of RIGL§ 16.22.30-33 with regard to adopting high-quality curriculum and includes a list of approved curricula for ELA and Mathematics.

Curriculum Used in Rhode Island: This list and visualization displays which K–12 curricula are being used in each LEA and designates their quality as either *red*, *yellow*, *green*, *not yet rated*, or *locally developed*.

Additional Resources

Tools for Reviewing Curriculum

Resource	Description
EdReports	The EdReports curriculum reports are the primary source for rating mathematics curriculum in Rhode Island as high quality.
Math Evidence for ESSA	Created and managed by Johns Hopkins University, Evidence for ESSA rates mathematics programs on their degree of effectiveness and their alignment to the standards set down by ESSA. Many of the programs rated here are supplemental as opposed to comprehensive curricula.
What Works Clearinghouse	Reviews existing research on mathematics programs and products to answer the question, “What works in education?”
WIDA Prime V2	A tool to assist publishers and educators in analyzing their materials for the presence of key components of the WIDA Standards Framework. PRIME stands for Protocol for Review of Instructional Materials for ELLs. Although not keyed to the 2020 WIDA Standards, it still offers invaluable information.
High-Quality Curriculum Selection and Implementation: Understanding Culturally Responsive and Sustaining Education (CRSE) in Math	This template guides a user through such considerations as cultural awareness and critical consciousness when working through the adoption process.
Multilingual Learner (MLL) Non-Negotiables for Math Curriculum Selection	The purpose of this template is to prompt adopters to consider the necessary opportunities for language development, access to grade-level content, and the integration of scaffolding within a curriculum.

Curriculum Selection Resources

Resource	Description
Selecting for Quality: A Guide for Adopting High-Quality Instructional Materials	This step-by-step template, created by EdReports, is intended to guide LEAs through the adoption process.
Additional Review Tools to Support the Selection of a High-Quality Curriculum in RI: A Guidance Document	The document includes guidance on selecting curriculum through the lens of culturally responsive and sustaining education and multilingual learner supports.
Instruction Partners Curriculum Support Guide	The guide includes a workbook designed to be used by teams of educators working together through the selection and implementation journey for their school or system.

References

The Center on Standards and Assessment Implementation (CSAI). (2018). *CSAI Update: Standards Alignment to Curriculum and Assessment.* https://files.eric.ed.gov/fulltext/ED588503.pdf

Common Core State Standards Initiative. (2020). *Key Shifts in Mathematics*. http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

Massachusetts Department of Elementary and Secondary Education (MDOE). (2017). *Mathematics Curriculum Framework – 2017.* https://www.doe.mass.edu/frameworks/math/2017-06.pdf

National Governors Association (NGA) Center for Best Practices, Council of Chief State School Officers (CCSSO). (2010). *Common Core State Standards for Mathematics (CCSSM)*. Washington D.C. Retrieved from http://www.corestandards.org/read-the-standards/

Section 3: Implementing High-Quality Instruction

Part 1: Introduction and Overview

As described in Sections 1 and 2 of this framework, while robust standards and high-quality curriculum materials (HQCMs) are essential to providing all students the opportunities to learn what they need for success in college and a career of their choosing, high-quality instruction is also needed. Standards define what students should know and be able to do. HQCMs that are aligned to the standards provide educators with a roadmap and tools for how students can acquire that knowledge and skill. It is high-quality instruction that makes the curriculum come alive for students. High-quality instruction gives all students access and opportunity for acquiring the knowledge and skills defined by the standards with a culturally responsive and sustaining approach. "When teachers have great instructional materials, they can focus their time, energy, and creativity on meeting the diverse needs of students and helping them all learn and grow." (Instruction Partners Curriculum Support Guide Executive Summary, p. 2) Executive-Summary-1.pdf (curriculumsupport.org)

The process of translating a high-quality curriculum into high-quality instruction involves much more than opening a box and diving in. This is because no single set of materials can be a perfect match for the needs of all the students that educators will be responsible for teaching. Therefore, educators must intentionally plan an implementation strategy in order to have the ability to translate HQCMs into high-quality instruction. Some key features to attend to include:

- Set systemic goals for curriculum implementation and establish a plan to monitor progress,
- Determine expectations for educator use of HQCMs,
- Craft meaningful opportunities for curriculum-based embedded professional learning,
- Factor in the need for collaborative planning and coaching (Instruction Partners Curriculum Support Guide Executive Summary, page 4) Executive-Summary-1.pdf (curriculumsupport.org), and
- Develop systems for collaboratively aligning HQCMs to the WIDA ELD Standards.

Thus, with a coherent system in place to support curriculum use, teachers will be well-positioned to attend to the nuances of their methods and make learning relevant and engaging for the diverse interests and needs of their students.

Given the above, what constitutes high-quality instruction? In short, high-quality instruction is defined by the practices that research and evidence have demonstrated over time as the most effective in supporting student learning. In other words, when teaching is high quality it embodies what the field of education has found to work the best. Therefore, this section provides a synthesis of research- and evidence-based practices that RIDE believes characterizes high-quality instruction in *mathematics*. This section begins by describing the high-quality instructional practices that apply across content areas and grades with details and examples that explain what these instructional practices look like in mathematics, and later explains other specific instructional practices that are at the core of high-quality instruction in mathematics. The instructional practices articulated in this section are aligned with and guided by best practices for multilingual learners and for differently-abled students, and specific information and resources are provided about how to support all students in their learning while drawing on their individual strengths. These instructional practices also contribute to a multi-tiered system of support (MTSS) in which *all* students have equitable access to strong, effective core instruction that supports their academic, behavioral, and social-emotional outcomes. This section on instruction ends with a set of resources and tools that can facilitate high-quality instruction and professional learning about high-quality instruction, including tools that are relevant across content areas and grade levels and those that are specific to mathematics.

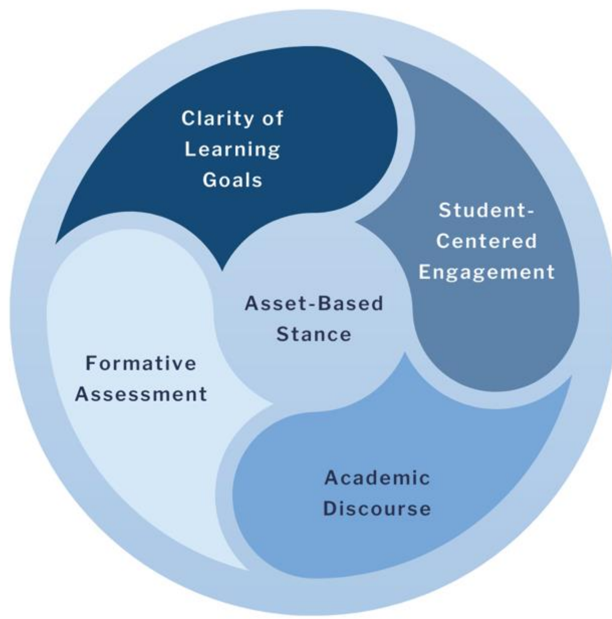
In reviewing this section, use Part 2 to understand what high-quality instruction should look like for all students in mathematics. Use Part 3 to identify resources that can promote and build high-quality instruction, as well as resources for learning more about how to enact high-quality instruction.

Part 2: High-Quality Instructional Practices

In order to effectively implement high-quality curriculum materials, as well as ensure that all students have equitable opportunities to learn and prosper, it is essential that teachers are familiar with and routinely use instructional practices and methods that are research- and evidence-based. Below are instructional practices that are essential to effective teaching and learning in mathematics. The first set of instructional practices are those common across all disciplines and curriculum frameworks. These are followed by instructional practices specific to mathematics. For additional guidance, there are also descriptions and references to instructional practices that support specific student groups, such as multilingual learners and differently-abled students.

High-Quality Instruction in All Disciplines

Below are five high-quality instructional practices that RIDE has identified as essential to the effective implementation of standards and high-quality curriculum in all content areas (See figure to the right). These practices are emphasized across all the curriculum frameworks and are supported by the design of the HQCMs. They also strongly align with the instructional framework for multilingual learners, the [high-leverage practices (HLPs) for students with disabilities](), and RIDE's teacher evaluation system. Below is a brief description of each practice and what it looks like in mathematics.



Assets-Based Stance

Teachers routinely leverage students' strengths and assets by activating prior knowledge and connecting new learning to the culturally and linguistically diverse experiences of students while also respecting individual differences.

What This Looks Like in Mathematics

Taking an assets-based stance in mathematics instruction requires an educator to understand and acknowledge that all of their students can be competent learners and doers of mathematics. This begins with the recognition that each student brings with them a wealth of relevant knowledge that can serve as a bridge in refining and expanding their mathematical thinking and understanding. All students, regardless of their age, enter a classroom having experienced diverse quantitative situations, both in and outside of school, from which they have learned a collection of things about number (National Research Council, 2001). An assets-based stance challenges teachers to view their students' unique cultural and linguistic experiences as intellectual resources (Celedón-Pattichis et al., 2018).

Effective instruction makes use of students' unique quantitative understandings and cultural experiences by creating scaffolds between their prior knowledge and new knowledge. By providing students with the opportunity to engage with rigorous mathematical tasks, allowing students to share their mathematical ideas and thinking, acknowledging there are different ways to do mathematics, and connecting to prior knowledge, teachers empower students to be creative and powerful mathematical thinkers. All students should have routine and supported mathematical experiences that link to their identities and foster an understanding of both higher order concepts and foundational skills. If "teachers treat students as brilliant mathematical thinkers and expect students to demonstrate such mathematical brilliance in the classroom space (Jett, 2013)," they increase the likelihood of mathematical competence and empower students to continue to grow in their mathematical understanding (Gay, 2010; Ladson-Billings, 2009).

What this looks like in relation to Universal Design for Learning (UDL)

Differentiated core instruction based in UDL provides access and equity for each student providing multiple options for learning and expression without changing what is being taught. Differentiation is proactive with the goal of adjusting the *how*, based on understanding learner assets and needs, so students may achieve maximum academic growth. High-quality curriculum and instruction implemented through UDL and differentiation support access to grade-level curriculum as part of Tier 1 of a multi-tiered system of supports (MTSS).

What this looks like for Multilingual Learners (MLLs)

Educators with MLLs in their class will advance student learning by drawing on MLLs' home languages, lived experiences, and world views. Although RIDE encourages student use of academic registers, it is important that educators and administrators maintain an asset-oriented stance in facilitating academic discourse and student understanding of standard English conventions, particularly when working with learners from minoritized groups. Educational agencies can play a role in sustaining the linguistic traditions of their students. Thus, classroom discourse, when done well, will reflect the discourse practices of local communities—capturing the rich ways families actually use language, rather than making prescriptive judgements about how students and their families ought to talk.

What this looks like for Differently-Abled Students (DAS)

Implementation of HLP 3: Collaborate with Families to Support Student Learning and Secure Needed Services promotes an assets-based stance for students with IEPs. Effective collaboration between educators and families is built on positive interactions in which families and students are treated with dignity. Educators affirm student strengths and honor cultural diversity maintaining open lines of communication with phone calls or other media to build on students' assets and discuss supports or resources. Trust is established with communication for a variety of purposes and not just for formal reasons such as report cards, discipline reports, or parent conferences.

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
3 Steps to Developing an Asset-Based Approach to Teaching	**Article** on how to build upon what your students bring to the classroom

Resource	Description
Five Ways to Build an Asset-Based Mindset in Education Partnerships	**Article** on developing an asset-based mindset
An Asset-Based Approach to Support ELL Success	**Article** on strategies for engaging and supporting MLLs
HLP #3: Collaborate with Families to Support Student Learning and Secure Needed Services	**Leadership Guide** for HLP #3: Collaborate with Families to Support Student Learning and Secure Needed Services
Stories from the Classroom: Focusing on Strengths within Assessment and Instruction | Progress Center	**Video** from Progress Center on including students in examining their data and setting ambitious goals by focusing on their assets
TIES TIPS | Foundations of Inclusion | TIP #6: Using the Least Dangerous Assumption in Educational Decisions | Institute on Community Integration Publications (umn.edu)	**Article** on how the least dangerous assumption pushes educators to consider all students as capable. The challenge is to replace a deficit mindset and consider what can educators do to support students in how they access, engage in, and respond not only to both academic and life skills content
Beyond IEPs and 504 Plans: Why You Should Consider Asset-Based Accommodations	**Article** on how asset-based accommodations beyond IEPs and 504s can be effective tools for supporting academic achievement and future success
Classroom Supports: Universal Design for Learning, Differentiated Instruction CTE Series 3 | NTACT:C (transitionta.org)	**Webinar** on the CAST framework of UDL and explanations for how one district incorporates UDL into their CTE programs
MTSS for All: Including Students with the Most Significant Cognitive Disabilities	**Brief** from the TIES Center that provides suggestions for ways in which MTSS can include students with the most significant cognitive disabilities

Clear Learning Goals

Teachers routinely use a variety of strategies to ensure that students understand the following:

1. *What they are learning* (and what proficient work looks like),
2. *Why they are learning it* (how it connects to what their own learning goals, what they have already learned and what they will learn), and
3. *How they will know when they have learned it*.

What This Looks Like in Mathematics

Establishing clear goals and expectations linked to the standards, high-quality curriculum, and learning trajectories or progressions serve to guide teachers' decision-making and focus students' attention during a lesson. Moreover, they assist the student in monitoring their own progress in learning the mathematics (NCTM, 2014). Studies indicate students perform at higher levels when expectations for learning are clear (Haystead & Marzano, 2009; Hattie, 2009). Key questions for both teachers and students to consider with respect to goals are:

- What mathematics content is being taught and what mathematical practices will support the learning?
- Why is the mathematics important — both for school and for life?
- How does the mathematics relate to what has already been learned — both in school and from lived experiences?
- Where are these mathematical ideas going — how will they help with future understandings? (NCTM, 2014)

Part of clarifying learning goals for students is to communicate criteria for proficiency and to model for students a process to self-assess their progress in meeting the criteria. With guidance and practice, students can compare their own understanding and work to the established criteria in order to identify their successes, be mindful of areas of improvement, and engage in strategies to grow their understanding to a level of proficiency (New South Wales Government – Education, 2021).

What this looks like for Multilingual Learners (MLLs)

For educators with one or more active MLLs on their roster, clear learning goals for MLLs will consist of explicit language goals to guide instruction in mathematics. Educators will model effective use of disciplinary academic vocabulary and syntax, creating opportunities every day for explicit disciplinary language development, aligned to the WIDA ELD Standards.

What this looks like for Differently-Abled Students (DAS)

HLP 14, Teach Cognitive and Metacognitive Strategies to Support Learning and Independence, supports the high quality instruction practice of Clear Learning Goals. Through task analysis, educators can support DAS by determining the steps they need to take to accomplish goals, then create and teach a procedure to help the student meet the goals. The educator uses explicit instruction (HLP 16) to teach the student self-regulation strategies such as self-monitoring, self-talk, goal-setting, etc. Clear, step-by-step modeling with ample opportunities for practice and prompt feedback coupled with positive reinforcement (HLP 22) in different contexts over time ensure that DAS become fluent users of metacognitive strategies toward understanding and achieving learning goals.

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
High-Leverage Practice (HLP) Leadership Guides from the Council for Exceptional Children	**Leadership Guides** for the following HLPs: #11: Identify and Prioritize Long- and Short-Term Learning Goals #12: Systematically Design Instruction Toward Learning Goals #13: Adapt Curriculum Materials and Tasks #14: Teach Cognitive and Metacognitive Strategies to Support Learning and Independence #16: Use Explicit Instruction

	#22: Provide Positive and Constructive Feedback to Guide Students' Learning and Behavior (academic)
High-Leverage Practice Videos for HLP #11 and HLP #16	**Videos** highlighting **HLP #11** (identify and prioritize long- and short-term learning goals) and **HLP #16** (use explicit instruction) found under "**Access Videos.**"
Culturally Responsive Teaching for Multilingual Learners: Tools for Equity	**Videos** to support culturally responsive and sustaining teaching that showcase strategies, such as activating background knowledge and partnering with MLL families.
Stories from the Classroom: Focusing on Strengths within Assessment and Instruction | Progress Center	**Video** from Progress Center on including students in examining their data and setting ambitious goals
Intensive Intervention Course Content: Features of Explicit Instruction | National Center on Intensive Intervention	**Course content** to support educators in providing explicit instruction in whole groups or small groups

Student-Centered Engagement

Teachers routinely use techniques that are student-centered and foster high levels of engagement through individual and collaborative sense-making activities that promote practice, application in increasingly sophisticated settings and contexts, and metacognitive reflection.

What This Looks Like in Mathematics

Today's mathematics classroom should be a vibrant space in which students take charge of their own learning, think deeply, communicate and collaborate with their peers, and persist to solve meaningful problems. The Standards for Mathematical Practice (MPs) clearly articulate these behaviors and others that students should engage in while learning mathematics from kindergarten through high school. Quality mathematics instruction grounded in the practices facilitates dynamic learning experiences for students, discouraging passivity and encouraging student agency. Learning is optimized when students engage in the mathematical practices while solving rigorous tasks requiring reasoning (MP2) and higher order thinking skills. Teachers should select high-quality tasks that offer multiple points of entry, allow for a wide array of solution strategies, and connect to students' current level of understanding, prior knowledge, and lived experiences. When students are able to gain access to these tasks, the mathematics at hand becomes meaningful. The potential to expand or solidify a student's current level of understanding is increased. Additionally, intentional and repeated exposure to higher level tasks provides students with insight into how long and hard (MP1) they may need to work at a task and an implicit understanding of how mathematics works (NCTM, 2014).

Engagement with rigorous mathematical tasks requires deep thinking and reasoning on the part of the student. As educators, it is necessary to overcome the desire to "rescue" students when they become frustrated with a task. Avoid the inclination to stop at the first sign of frustration and walk students through tasks step by step (Reinhart, 2000). Instead, during the planning process, consider where frustration and missteps may occur and intentionally design strategies to support students' sense-making and engagement while maintaining the rigor of a task (NCTM, 2014). Encourage students to take responsibility for their learning by having them ask questions of both you and their peers (MP3). Help them discover entry points into a problem by probing their thinking, thus giving them license to make mistakes and connections to content and relationships they already understand. Foster the mindset that making sense of mathematics is not a product of innate ability, but rather one of hard work and perseverance (Dweck, 2008). By allowing students to engage in productive struggle, they become empowered mathematical learners and deeper mathematical thinkers.

What this looks like for Multilingual Learners (MLLs)

Educators with MLLs in their class can promote student-centered engagement by providing scaffolded opportunities for students to build conceptual understanding and fluency with core disciplinary skills, appropriate to their English language proficiency levels. Home language materials and instruction are particularly powerful in promoting student-centered engagement with MLLs.

What this looks like for Differently-Abled Students (DAS)

Student-centered engagement is maximized when educators implement HLP 7, Establish a Consistent, Organized, and Respectful Learning Environment. DAS benefit from educators who explicitly teach consistent classroom procedures and expected behaviors while considering student input. Viewing behavior as communication, reteaching expectations and procedures across different school environments, and helping students understand the rationale for the rules and procedures as part of HLP 7 implementation will enhance student-centered engagement for DAS. In any content area, this may mean providing additional opportunities to for DAS to learn and practice routines that some peers might already have mastered. Some IEPs may call for self-monitoring checklists and visual schedules to support students in active participation in learning activities. Individual DAS will need specific supports unique to their learning profiles. Educators can implement HLP 7 in conjunction with HLP 18, Use Strategies to Promote Active Student Engagement, and HLP 8, Provide Positive and Constructive Feedback to Guide Students' Learning and Behavior, for individualized student supports.

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
High-Leverage Practice (HLP) Leadership Guides from the Council for Exceptional Children	**Leadership Guides** for the following HLPs: #7: Establish a Consistent, Organized, and Respectful Learning Environment #8: Provide Positive and Constructive Feedback to Guide Students' Learning and Behavior #17: Use Flexible Groupings #18: Use Strategies to Promote Active Student Engagement #21: Teach Students to Maintain and Generalize New Learning Across Time and Settings
Fundamental Skill Sheets Videos	**Video playlist** from the Iris Center: Choice Making, Proximity Control, Wait Time, Behavior Specific Praise **Note:** Video 6 focuses on proximity control in a high school mathematics class. The mnemonic used in the lesson is employed as a support after initial instruction that focused on developing conceptual

Resource	Description
	understanding of the trigonometric functions and their relationship to right triangles.
High-Leverage Practices Video: Use Strategies to Promote Active Student Engagement	**Video** highlighting HLP #18 which focuses on strategies to promote active student engagement
HLP #21: Teach Students to Maintain and Generalize New Learning Across Time and Settings	**Leadership Guide** for HLP #21: Teach Students to Maintain and Generalize New Learning Across Time and Settings
Including Voice in Education: Addressing Equity Through Student and Family Voice in Classroom Learning	**Infographic** on incorporating student voice and/or family voice into student learning, a promising strategy for teachers striving to foster culturally responsive and sustaining classrooms to enhance education access, opportunity, and success for students who are historically marginalized within the pre-kindergarten to grade 12 education systems
SEL for Self-Management	**RIDE resources** on Social Emotional Learning Indicators for Self-Management
SEL for Social Awareness	**RIDE resources** on Social Emotional Learning Indicators for Social Awareness
WWC | Organizing Instruction and Study to Improve Student Learning (ed.gov)	**Guide** including a set of concrete actions relating to the use of instructional and study time that are applicable to subjects that demand a great deal of content learning, including social studies, science, and mathematics. The guide was developed with some of the most important principles to emerge from research on learning and memory in mind. • Space learning over time. • Interleave worked example solutions with problem-solving exercises. • Combine graphics with verbal descriptions. • Connect and integrate abstract and concrete representations of concepts. • Use quizzing to promote learning. Use quizzes to re-expose students to key content. • Ask deep explanatory questions.

Academic Discourse

Teachers routinely facilitate and encourage student use of academic discourse through effective and purposeful questioning and discussion techniques that foster rich peer-to-peer interactions and the integration of discipline-specific language into all aspects of learning.

What This Looks Like in Mathematics

The Rhode Island Professional Teaching Standards (RIDE, 2007, October) establish the expectation that teachers create a learning environment that is safe, secure, nurturing, and supportive of active engagement by all students. In this environment, mutual respect and intellectual risk-taking are modeled by the teacher and expected of all. Students are given opportunities to work both independently and collaboratively, and to take responsibility for their own learning. When these conditions are present in the mathematics classroom, the situation is ideal for the teacher to facilitate meaningful mathematical discourse.

The third Standard for Mathematical Practice calls for students to "construct viable arguments and critique the reasoning of others" (NGA Center & CCSSO, 2010). Classrooms that strategically promote student-to-student discourse in addition to student-to-teacher discourse provide the environment in which students can develop and nurture this skill. When planned effectively, mathematical discussions in the classroom promote an exchange of ideas that can clarify understanding, develop the use of academic language, enable students to see the mathematics from a variety of vantage points, and advance the conceptual understanding of all students. When students feel safe to share their ideas, their understandings, and their misunderstandings, they are better positioned to take charge of their own learning and to support the understanding of their peers.

It is important for the teacher, as facilitator, to set the stage for meaningful discourse and to have a variety of routines to stimulate participation and support communication for all. One routine suggested by Smith and Stein (2011) outlines these considerations:

- *Anticipate* student responses prior to a lesson.
- *Monitor* students' work and task engagement.
- *Select* specific students to present their mathematical work.
- *Sequence* students' responses in a specific order during discussion.
- *Connect* the different student responses to one another and to the key mathematical ideas of the lesson.

By following these key practices, discourse will remain focused on the goal of the lesson, build upon different approaches to addressing the problem of the lesson, and maximize access to understanding the content of the lesson for all students.

What this looks like in relation to Social Emotional Learning

The five core competencies of Rhode Island's Social Emotional Learning standards and indicators support academic discourse across the content areas. Learners must engage effectively in a range of collaborative discussions with diverse partners, building on each other's' ideas and expressing their own clearly.

- Self-Awareness: Identifying one's strengths and weaknesses while working within a group, staying motivated and engaged throughout the work.
- Self-Management: Controlling one's emotions, responding calmly to comments, questions, and nonverbal communication.
- Social-Awareness: Understanding others' perspectives and cultures, compromising with peers when the situation calls for it, accepting feedback from peers and teachers, listening to the opinions of others and taking them into consideration.

- Relationship Skills: Expressing one's perspective clearly, following agreed upon rules of the group and carrying out assigned role(s), gaining peers' attention in an appropriate manner, asking questions of group members, limiting the amount of information shared with others, and actively listening to peers when they speak.
- Responsible Decision Making: Coming to the group prepared, demonstrating independence with work tasks, dividing labor to achieve the overall group goal efficiently.

Social and emotional skills are implicitly embedded in the content standards, and students must learn many social and emotional competencies to successfully progress academically. Social Emotional Learning skills are instrumental for each student and are crucial for differently-abled students.

What this looks like for Multilingual Learners (MLLs)

Though beneficial for all students, academic discourse is especially important for MLLs because engaging in authentic interaction with discipline-specific oral language facilitates MLLs' overall development of English language proficiency. In RIDE's *High-Quality Instructional Framework for MLLs to Thrive*, academic discourse is defined as a sustained spoken interaction between two or more students in which knowledge is shared using the conventions of particular genres and disciplines.

What this looks like for Differently-Abled Students (DAS)

Educators plan mixed-ability small groups to increase DAS student engagement in academic discourse through a variety of cooperative learning structures consistent with HLP 17, Use Flexible Groupings. Effective groupings are monitored for learning and student interactions to meet various academic, behavioral, and interpersonal instructional objectives. DAS may require varied group sizes and types based upon specific IEP goals and accommodations. A student engaging in intensive instruction of a particular math or reading skill may do so in a supplemental homogenous group of only 2-3 peers while also having regular opportunities to engage in heterogeneous collaborative groups during core instruction with scaffolded supports.

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
High-Leverage Practice (HLP) Leadership Guides from the Council for Exceptional Children	**Leadership Guides** for the following HLPs: #15: Provide Scaffolded Supports #17: Use Flexible Groupings
Instruction | High-Leverage Practices	**Resources for high-leverage practices related to instruction.** When instruction is well designed, strategic, and adaptable, special education teachers have the skills to improve student learning.
TIES TIP #2: Using Collaborative Teams to Support Students with Significant Communication Needs in Inclusive Classrooms	**Tip sheet** on additional planning for general and special education teachers as well as related service providers. These include speech-language pathologists, physical and occupational therapists,

Resource	Description
	and vision/hearing specialists. Coordinating the work of these service providers and leveraging their expertise can result in a high-quality experience for all the learners in an inclusive class.
Doing and Talking Math and Science: Strengthening Reasoning, Strengthening Language	**Discourse Move Cards** for students and teachers in STEM classrooms

Formative Assessment

Teachers routinely use qualitative and quantitative assessment data (including student self-assessments) to analyze their teaching and student learning in order to provide timely and useful feedback to students and make necessary adjustments (e.g., adding or removing scaffolding and/or assistive technologies, identifying the need to provide intensive instruction) that improve student outcomes.

What This Looks Like in Mathematics

With goals and expectations established and clearly communicated to students, teachers should continually monitor and assess their students' understanding. Using a variety of assessment strategies, both formal and informal, teachers can use the associated data in a formative way to adjust instruction with the goal of improving student understanding, and not just to assign a grade. While well-designed tests and quizzes are two traditional sources for assessing student understanding, eliciting evidence of student thinking in a formative manner, integrated into instruction, has proven to be most efficacious in nimbly responding to student needs and improving student outcomes (NCTM, 2014).

Gathering evidence of student thinking requires significant planning and forethought. It is essential to align the goals of instruction with the evidence needed to measure attainment of those goals. Educators need to first identify indicators of what is important to take note of in students' mathematical thinking and then plan ways to make that thinking transparent. Questions, tasks, and activities should be constructed to lay bare students' patterns of reasoning, including missteps and misconceptions in that reasoning (NCTM, 2014). There should be "a constant stream of information about how student learning is evolving toward the desired goal" (Heritage, 2008).

Formulating and posing well-crafted questions is a powerful tool for eliciting evidence of student thinking, and there is abundant research on effective questioning techniques in mathematics. Generally speaking, research indicates the *type* of questions and the *patterns* in which those questions are posed should encourage students to both explain and reflect upon their thinking. There are times when teachers should pose questions to gather information, probe thinking, make the mathematics visible, and encourage reflection and justification. Each type of question elicits different types of information that are essential to enabling a teacher to assess and advance a student's learning (NCTM, 2014).

The *pattern of questioning* also contributes to students making sense of the mathematics. Questions should be focused to allow for students to clearly communicate their thoughts and rationale, and to foster reflection on those thoughts and those of their peers. The objective is not to get to a single predetermined answer, but rather to open a window to the world of student thinking, thus allowing the teacher to employ their content knowledge to respond to student needs (NCTM, 2014).

The process of eliciting data and adjusting instruction based on that data are just two parts of an overall strategy to improve student learning and assist them in becoming responsible and committed to that learning. Providing descriptive, targeted, and actionable feedback is the glue that cements these efforts into a meaningful and effective whole. The research of Clarke (2003) and Hattie and Timperley (2007) drives home the importance of providing feedback linked to effort and perseverance, identifying successes, and lending support and advice on how to improve (OER4 Schools, 2013).

Hattie and Timperley posit feedback should be targeted to the task, process, or self-regulatory, and not the personal level. "It is most effective when it aids in building cues and information regarding erroneous hypotheses and ideas and then leads to the development of more effective and efficient strategies for processing and understanding the material." Teachers and students must be adept at providing and reacting to feedback, respectively, thus necessitating educators and families coaching students in how to proactively respond to feedback (Hattie & Timperley, 2007).

What this looks like for Multilingual Learners (MLLs)

For educators with one or more active MLLs on their roster, formative assessment practices should include the collection of discipline-specific language samples and progress-monitoring of MLLs' language development in science. These language samples and assessment practices will give educators the data needed to provide students with language-focused feedback aligned to their language goals for science. When designing or amplifying formative assessments for disciplinary language development, educators should draw on the English language proficiency level descriptors for their grade level(s) in the WIDA ELD Standards Framework. For additional information about how these descriptors can assist educators in offering targeted feedback based on the word, sentence, or discourse level dimension of students' language samples, please see Section 4 of the Science Curriculum Framework.

What this looks like for Differently-Abled Students (DAS)

HLP 4, Use Multiple Sources of Information to Develop a Comprehensive Understanding of a Student's Strengths and Needs, describes assessment as a collaborative process that includes informal assessments to plan instruction that is responsive to individual needs. DAS participation in formative assessments may require specific accommodations specified in IEPs. Implemented in conjunction with HLP 22, Provide Positive and Constructive Feedback to Guide Students' Learning and Behavior, DAS will receive immediate and specific feedback on their performance that is goal-directed and thoughtful in considering the specific learner profile. Feedback on formative assessment is positive and constructive when it avoids words like "should, but, however" and includes statements that highlight what they did appropriately followed by a question (what is another way?) or a suggestion (try adding or continuing with). A diagram or image can support DAS to understand feedback and their progress on formative assessments.

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
CCSSO Revising the Definition of Formative Assessment	This resource provides an overview of the FAST SCASS's revised definition on formative assessment, originally published in 2006. The revised definition

Resource	Description
	includes an overview of the attributes of effective formative assessment and emphasizes new areas emerging from current research, theory, and practice.
High-Leverage Practice (HLP) Leadership Guides from the Council for Exceptional Children	**Leadership Guides** for the following HLPs: #4: Use Multiple Sources of Information to Develop a Comprehensive Understanding of a Student's Strengths and Needs #6: Use Student Assessment Data, Analyze Instructional Practices, and Make Necessary Adjustments that Improve Student Outcomes HLP #8: Provide Positive and Constructive Feedback to Guide Students' Learning and Behavior (SEL) #22: Provide Positive and Constructive Feedback to Guide Students' Learning and Behavior (academic)
High-Leverage Practices Video: Provide Positive and Constructive Feedback to Guide Students' Learning and Behavior	**Video** highlighting HLPs #8 and #22 on providing positive and constructive feedback to guide students' learning and behavior. This resource supports both SEL and academic domains.
Stories from the Classroom: Focusing on Strengths within Assessment and Instruction | Progress Center	**Video** from Progress Center on including students in examining their data and setting ambitious goals
Assessment | High-Leverage Practices	**Resources** for using multiple sources of assessment, communicating assessment data, and using data to inform instruction

High-Quality Instruction in Mathematics

Develop Procedural Skill and Fluency through Conceptual Understanding

Teachers routinely attend to building student fluency with procedures by grounding instruction in the conceptual underpinnings of those procedures. They provide varied types of practice through which students become flexible and efficient users of procedures when solving mathematical and real-world problems (NCTM, 2014).

The standards call for a balance of instructional attention between developing conceptual understanding, procedural skill and fluency, and the ability to apply mathematics when solving rigorous mathematical and real-world problems — the three elements of rigor identified by the standards. The goal of instructing students to gain fluency has an interesting history in the United States. Traditionally, American instruction has approached this goal only from a procedural vantage point. Classroom materials and instruction were characterized by step-by-step instructions leading to a specific outcome. If students were individually successful in gaining some insight into why a procedure or algorithm worked, all the better. Research has come to demonstrate that procedural skill and fluency is best developed with a sound grounding in conceptual understanding, strategic reasoning, and problem-solving (NCTM, 1989, 2000; NGA Center & CCSSO, 2010). When facts and procedures are connected to understanding, they are easier to use and to transfer to varied situations. A student who is fluent in some aspect of mathematics is one who is able to flexibly select a method or strategy to solve mathematical or real-world problems. There is an element of efficiency in the selection as well as the ability to explain or justify the selection and the process (NCTM, 2014).

With conceptual understanding in place, students are able to build and strengthen their fluency through practice. As with any facet of instruction, designing opportunities for student practice must be done intentionally and with a specific goal in mind. Giving students too many practice problems too soon is not an effective approach to building fluency. Small sets of problems, connected to conceptual understanding, distributed over a period of time, and accompanied by timely and meaningful feedback are best for building strong fluency with procedural skill (NCTM, 2014). Interestingly, another tactic for fluency practice, although frequently disregarded, is to use rich problem-solving tasks in which students need to employ specific computational skills while designing strategies to navigate the tasks. This so-called embedded practice can really pack a bang for the buck (National Research Council, 2001).

Use and Connect Multiple Representations

Teachers routinely incorporate the use of multiple representations into their mathematics instruction. They challenge their students to do the same knowing that when connections are made between different representations, there is a deepening of conceptual understanding and the ability to apply procedures when solving problems (NCTM, 2014).

Experts in mathematics education stress the power of multiple representations — physical, visual, contextual, verbal, and symbolic. When students are asked to use different representations in mathematics and talk about the similarities in those representations, they not only gain improved access to the mathematics at hand, but also develop a deeper understanding of that mathematics. Perhaps Tripathi (2008) expressed it best when saying, “Different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture (concept) richer and deeper.” Flexibility in the use of representations is linked to increased success in problem-solving and the ability to see and grapple with the essential structures in mathematics (NCTM, 2014).

Young students are no strangers to representing their mathematical thinking with diverse representations. They commonly use drawings or act out situations using manipulatives to gain entry into a problem and to formulate and justify a solution (National Research Council, 2001). These simple representations can serve as the connections to creating deeper mathematical understanding, evolving into efficient symbolic representations that can be justified and explained.

Visual and physical representations should not be the territory of solely the very young. It is interesting to note that visual representations are particularly helpful in increasing access to the mathematics for many learners. This is expressly true for differently-abled students and MLLs. Students’ visual representations tend to leave more overt clues to their thinking used when solving a problem and can serve as a platform for classroom discourse, allowing other students to gain access to and understanding of the mathematics (NCTM, 2014).

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
Instructional Approaches for Math Rigor	This chart created by Achievement Network identifies instructional strategies that are especially effective for addressing each aspect of rigor in the standards (conceptual understanding, procedural skill and fluency, and application).
NCTM Position: Procedural Fluency in Mathematics	This position paper by NCTM answers the question, “What is procedural fluency, and how do we help students develop it?”
Middle School Math Fluency Guide	This UnboundEd guide offers strategies and rationale for busy teachers who want to develop procedural fluency in middle school mathematics. Connect and build on students’ conceptual understanding through structured practice that will help students make sense and expertise with procedures. Learn how to effectively support student development of their productive disposition toward mathematics.

Evidence-Based Practices for Supporting Integration of STEM

RIDE is committed to increasing access to high-quality STEM educational opportunities for all students. An understanding of STEM concepts and development of STEM-related skills is needed to prepare future generations to make informed choices and increase the number of qualified candidates for careers in Rhode Island’s growing STEM industries.

The acronym, STEM, was coined by the National Science Foundation (NSF) in 2001 to describe occupations that required knowledge and skills from the disciplines of Science, Technology, Engineering, and Mathematics. Beyond conceptual understanding, STEM occupations require the application of concepts across disciplines. In 2010, the Rhode Island School of Design (RISD) campaigned to add Art and Design to the acronym by revising it to STEAM. This addition shifted the term to highlight the more innovative aspects of creativity and problem-solving. The U.S. Bureau of Labor Statistics anticipates the number of STEM occupations to grow an additional 8 percent by 2029, compared with 3.7 percent for non-STEM occupations in the same period. To ensure that students have the knowledge and skillset to be successful in STEM occupations, all students need to engage in STEM experiences that focus on application and problem-solving throughout their education. Engaging in well-designed, grade-level appropriate STEM activities from an early age will give all students experiences where they can apply concepts and skills acquired in core classes to develop innovative solutions to local and global problems.

The individual subject areas of science, technology, engineering, and mathematics are the disciplines of STEM, where a solid foundation is built. Building from this, students need to engage in *Integrated* STEM, where experiences apply the knowledge and skills from several (or all) of the STEM disciplines. Time for Integrated STEM should be provided beyond the time allotted for mathematics and science instruction since these subjects have tightly packed curricula that need to be followed with fidelity. An increasing number of schools have supplemental, in-school STEM/STEAM programs for elementary students and STEM/STEAM courses for middle and high school students. These in-school opportunities need to be part of every student’s experience, not just offered as electives or as enrichment for high achieving students.

Planning for STEM (or STEAM) Integration

- Integrated STEM experiences should support the development of disciplinary knowledge while making cross-discipline connections explicit to students. Educators must thoughtfully design Integrated STEM experiences that provide intentional support for students to build knowledge and skill both within the disciplines and across disciplines.
- Educators need to ensure that STEM experiences reinforce the student learning in science and mathematics, but do not undermine or duplicate the core subject curriculum.
- When designing Integrated STEM experiences, it is important to use the grade-level science and mathematics standards and learning progressions. Additionally, the Standards for Technology and Engineering Literacy (STEL) (ITEEA, 2020) should be used to guide to ensure that the experiences are appropriate for the developmental level of the students and develop students' technology and engineering proficiencies.
- Instructional models such as project-based/problem-based learning provide authentic opportunities for students to engage in Integrated STEM. Educators should design experiences that are grade-level appropriate and draw on student and/or community interest. The learning experiences should be iterative, annually reviewing them to incorporate new ideas or technology and to include novel student interests or community concerns.
- Integrated STEM education should not take the place of high-quality education focused on the individual STEM subjects, but it should require students to apply the knowledge and skills of the STEM subjects. While teachers should integrate STEM into math and science courses where it naturally fits, students need more opportunities to engage in Integrated STEM in school. Since the Next Generation Science Standards (NGSS) include engineering performance expectations as well as the practice of analyzing and interpreting data, there is some expected integration of the other STEM disciplines. Additional opportunities to engage in Integrated STEM will give students motivation to apply what they are learning in STEM discipline areas and advance their learning.

Real-World & Career Connections

- All students should view a career in STEM as accessible; engaging all students in STEM throughout K–12 is an important part of creating this perspective. Providing access to STEM experiences where students are challenged but can find success can lead to an interest in STEM careers. Schools and educators need create a climate that provides all students, especially those from underrepresented groups in STEM career fields, access and the opportunity to be successful in STEM learning. Partnering with local STEM organizations and industries will allow students to better understand the opportunities that exist through interaction with STEM professionals, exploration of potential careers, and understanding the variety of STEM-related workplaces. Industry partnerships can start at an early age as part of career awareness, later progressing to career exploration, and potentially including high school internships or pre-graduation training programs.
- Even if students do not follow a STEM career path, they will still need to acquire STEM literacy. STEM literacy includes the ability to be a critical consumer of information, be a creative problem solver, and develop critical thinking skills. Thoughtfully designed Integrated STEM experiences also build the Cross-Curricular Proficiencies of collaboration, communication, problem-solving and critical thinking, reflections and evaluation, and

research. These skills will support all students to be lifelong learners and have success in college and career.

Equitable Access to STEM

- Assuring access to STEM experiences for learners traditionally underrepresented in STEM fields can provide opportunities for individual success as well as broader changes to the STEM workforce. Additionally, engaging learners in STEM-related problem-based learning provides motivation and engagement not found in decontextualized academic knowledge. (Parker et al, 2016)
- Access is only one aspect of equity, schools also need to look carefully at how their designs and strategies encourage broadened participation through alternative ways of thinking about motivation, engagement, and persistence. Equity needs to be addressed with targeted strategies that align with the local context and realities of the learners, whether geographic (e.g., experiences of rural learners), social (e.g., experiences of girls), or experiential (e.g., experiences of students with disabilities). At the same time, strategies that are explicitly aligned to broadening participation in STEM also improve STEM experiences for all students. (Parker et al, 2016)

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
My PBLWorks from Buck Institute	To help schools and districts visualize high-quality PBL in the classroom, the Buck Institute for Education (BIE) has videos showcasing PBL projects from K–12 schools nationwide, including several STEM-themed projects. Teachers can view videos of successful PBL projects that feature teacher interviews and actual classroom footage and highlight projects from a range of grade levels, settings, and subject areas, including STEM.
STEMWorks at WestEd	STEMworks is a searchable online honor roll of high-quality science, technology, engineering, and mathematics (STEM) education programs. STEMworks helps companies, states, and individuals make smart investments in their communities by evaluating and cataloging programs that meet rigorous and results-driven design principles.
National Science Foundation (NSF) Resources for STEM Education	NSF research and development projects have produced a rich array of principles, materials, and practitioner insights that are helpful guides to improved preparation and professional development of STEM teachers. The following examples illustrate the range of ideas and products available from that work.

High-Quality Instruction for Multilingual Learners

The development of a second, third, or fourth language is a lifelong process — one that cannot take place in isolation or within a stand-alone hour of the school day. If we are to ensure all students have meaningful access to core instructional programs, all educators must share responsibility for the education of MLLs, including teachers of ELA/Literacy. For those not certified in English to Speakers of Other Languages or Bilingual/Dual Language, shared responsibility might beg the question: What is high-quality instruction for MLLs? What practices are evidence-based in promoting content and language learning with MLLs?

RIDE offers in-depth guidance about the key components of high-quality MLL instruction in its *High-Quality Instructional Framework for MLLs to Thrive*, but the research is clear: language development is most effective when integrated within content area instruction. Integrated language and content teaching gives MLLs rich, highly contextualized opportunities to use disciplinary language, which in turn reinforces content learning. Rather than teaching a discrete set of grammar rules or vocabulary lists, devoid of disciplinary context, educators must reflect on the language demands of content-based tasks from the core curriculum, offering explicit language instruction and ample scaffolds so MLLs can linguistically access and engage in core content area instruction.

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
WIDA Focus Bulletin- Collaboration: Working Together to Serve MLLs	**Article** with overview of language-focused collaborative teaching models and cycles
Professional Learning: Purposeful Instructional Design Part 1 Professional Learning: Purposeful Instructional Design Part 2	**Self-paced courses** on designing asset-based core instruction for MLLs This two-part course sequence is available on BRIDGE-RI, the learning management system for Multi-Tiered System of Supports (MTSS) in the state of Rhode Island. Educators can participate in these professional learning opportunities online at no cost. Critical aspects of Part 1 include: Tier 1 instructional design, data collection, and use of evidence-based instructional delivery practices for language learners, such as scaffolds. Critical aspects of Part 2 include: the role of physical environment and classroom climate in teaching and learning as well as translanguaging strategies and cross-linguistic features of common home languages.
Professional Learning Communities Facilitator's Guide for the What Works Clearinghouse Practice Guide Teaching Academic Content and Literacy to English Learners in Elementary and Middle School	**Videos** and **Facilitator's Guide** for four evidence-based practices: promoting academic vocabulary, integrating language and content instruction, providing structured opportunities to engage in writing activities, and conducting small-group interventions.

Resource	Description
The GO TO Strategies: Scaffolding Options for Teachers of English Language Learners, K-12	**Implementation Guide** for educators with a list of scaffolding strategies for MLLs
Focusing Formative Assessment on the Needs of English Language Learners	**Article** about conducting formative assessments with MLLs
Using Formative Assessment to Help English Language Learners	**Article** about conducting formative assessments with MLLs

High-Quality Instruction for Differently-Abled Students

Equity requires participation and a sense of belonging. To ensure that all students participate in science instruction — not just the hand raisers — teachers will need a continuum of proactive strategies that increase opportunities for student engagement. Students with IEPs or a 504 plan are general education students who access the grade-level curriculum through the support of high-quality instruction, as described in the preceding sections, which utilizes data on learner characteristics to differentiate and scaffold. Accommodations determined by the IEP team or a 504 plan complement the differentiation and scaffolds to ensure that accessibility needs specific to the individual learner are met. General education and content area teachers are responsible for providing instruction that is differentiated, scaffolded, and where appropriate for individual learners, includes accommodations. Some learners will also require instructional modifications as determined by the IEP team. When students receive quality supplementary curricula as part of their specially designed instruction (SDI), then inclusion can provide accommodations for generalizing skills they mastered in SDI. Collaborative planning with special educators and related service providers will support general educators in developing their repertoire of rigorous and accessible instructional practices.

The Leadership Implementation Guides from the High Leverage Practices for Students with Disabilities include tips for school leaders to support teachers; questions to prompt discussion, self-reflection and observer feedback; observable behaviors for teachers implementing the HLPs; and references and additional resources on each HLP. These guides, referenced throughout this section, were developed to help leaders integrate the HLPs into professional development and observation feedback.

Understanding learner characteristics will help clarify what types of support to provide to DAS in their planning, organizing, and writing to promote DAS access and progress in the science curriculum. A combination of techniques such as guided inquiry, science notebooks and Self-Regulated Strategy Development (SRSD) provides scaffolding to promote the success of DAS. Any accommodations outlined per the IEP or a 504 plan that provide reading, writing, and math access will be important for science (Collins & Fulton, 2017).

To Learn More

Below is a variety of links to resources to learn more about this practice.

Resource	Description
High-Leverage Practice (HLP) Leadership Guides from the Council for Exceptional Children	**Leadership Guides** for the following HLPs: #1: Collaborate with Professionals to Increase Student Success #5: Interpret and Communicate Assessment Information with Stakeholders to Collaboratively Design and Implement Educational Programs #14: Teach Cognitive and Metacognitive Strategies to Support Learning and Independence
Unit Co-Planning for Academic and College and Career Readiness in Inclusive Secondary Classrooms	**Article** describing the UCPG, a tool to support general and special education teacher collaboration and planning in inclusive general education classrooms
Big Ideas in Special Education: Specially Designed Instruction, High-Leverage Practices, Explicit Instruction, and Intensive Instruction	**Article** describing the differences between specially designed instruction, high-leverage practices, explicit instruction, and intensive instruction
IEP Tip Sheet: What are Supplementary Aids & Services?	**Tip Sheet** from Progress Center on accommodations for instruction and assessment, modifications, and other aids and services
IEP Tip Sheet: What are Program Modifications & Supports?	**Tip Sheet** from Progress Center on program modifications and supports that promote access to and progress in general education programming and shares tips for implementation
Can you implement DBI to support students with intellectual and developmental disabilities?	In this brief **video**, Dr. Chris Lemons shares considerations for implementing data-based individualization (DBI) to support students with intellectual and developmental disabilities
Classroom Supports: Universal Design for Learning, Differentiated Instruction CTE Series 3 | NTACT:C (transitionta.org)	**Webinar** from the National Assistance Center on Transition — UDL at secondary: "Fundamentals of differentiated instruction to support effective teaching, individualized learning and maximize student engagement are shared."
TIES Center: Inclusive Instruction: Resources on Inclusive Instruction	**Resources on Inclusive Instruction:** TIES Brief #4: Providing Meaningful General Education Curriculum Access to Students with Significant Cognitive Disabilities TIES Brief #5: The General Education Curriculum- Not an Alternative Curriculum! Lessons for All: The 5-15-45 Tool

Resource	Description
[TIES Center: TIES TIPS: Foundation of Inclusion TIPS]()	**TIES Inclusive Practice Series TIPS** #15 Turn and Talk in the Inclusive Classroom #16 Making Inferences in the Inclusive Classroom #19 Creating Accessible Grade-level Texts for Students with Cognitive Disabilities in Inclusive Classrooms
[Evidence-based practices for children, youth, and young adults with Autism]()	**Report** on evidence-based practice including a fact sheet for each that provides a longer description, information about participant ages and positive outcomes, and a full reference list.

Part 3: Resources for Professional Learning

Enacting the high-quality instructional practices described above is an essential yet complex task for teachers. Thus, ensuring high-quality instruction for all students in school often requires a team effort involving grade-level/content-area teachers, specialists and educators working with multilingual learners and differently-abled students in particular, and the administrators, leaders, and coaches who support all the educators. In addition, effective professional learning that helps teachers enhance their knowledge and application of high-quality instructional practices should strategically integrate multiple types of professional learning, as described in this section.

First, as mentioned in earlier sections of this framework, high-quality instruction begins with a deep understanding of the standards since they provide the foundation for instruction by defining what students need to know and be able to do. Professional learning suggestions and guidance for deepening the understanding of standards can be found in Section 2 of this framework.

Professional learning for high-quality instruction must also focus on developing a solid understanding of the high-quality instructional practices listed above. Readers are encouraged to review the many resources listed with each instructional practice and to establish ‘book study’ groups with colleagues to read, review and discuss any of the resources shared in Part 2 of this section of the framework.

In addition, supporting effective professional learning requires supporting teachers’ application of the practices described above. As with any complex skill, when supporting the application of high-quality instructional practices, the key ingredient is timely and targeted feedback. For feedback to be provided in a targeted and timely fashion, practices must be made visible so that the application of instructional practices can be observed. Once observed, feedback can then be generated. Most of the professional learning tools designed to provide feedback align with three key phases of the instructional cycle where it is very helpful for teachers to receive feedback about their instruction. The first phase is during lesson planning, before instruction actually takes place. The next phase is the actual instruction where teachers can be observed engaging with students. The final phase is after teaching has taken place and focused on the review of student work and evidence of learning. Below is a variety of tools and resources that are designed to provide teachers with feedback during these three phases. They are organized into the following three categories: Planning Tools, Observation Tools, and Evidence of Learning Tools. These tools come from a variety of sources, but all are intended to guide coaches, professional learning providers, and other leaders in offering support to teachers in this work.

Planning Tools

Resource	Description
30-Minute Tuning Protocol	**Protocol** designed to be used within collaborative teacher teams. It can be used to provide teachers with feedback on any artifact of their teaching and is a great tool to solicit feedback about lessons. In the protocol, a presenting teacher shares the goal, need, and plan of their professional work. Participants share feedback in rounds. The presenter then reflects on what was said that was helpful and what feedback they will try to incorporate to improve their plan.
UDL Tip for Designing Learning	**Tip sheet** with teacher questions, examples, and further resources to help anticipate learner variability and make instruction flexible and useful for all learners
CAST | Key Questions to Consider When Planning Lessons	**One-pager** of question prompts for teacher to improve lesson accessibility
Whole-Group Response Strategies to Promote Student Engagement in Inclusive Classrooms	**Article** on whole-group response systems paired with formative assessment charts to provide instruction that actively engages students in the learning process "These strategies can be implemented easily in classrooms with minimal additional resources and are applicable across grade levels and content areas with appropriate modifications."
Approaching Explicit Instruction Within a Universal Design for Learning Framework (See references section)	**Article** on implementation suggestions for using EL and UDL in tandem to better support students access and understanding of lesson content with improved student engagement and demonstration of what they know and can do
Achieve the Core's Lesson Planning and Reflection: Quick Reference Question Guide	The questions in this resource support a thoughtful planning and reflection process for mathematics instruction.
Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades	This practice guide, developed by the What Works Clearinghouse™ (WWC) in conjunction with an expert panel, distills contemporary mathematics intervention research into easily comprehensible and practical recommendations for teachers to use when teaching elementary students in intervention settings.

Resource	Description
Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students	This What Works Clearinghouse™ (WWC) practice guide presents evidenced-based suggestions for how to improve algebra skills and knowledge for students in grades 6–12. The guide offers three recommendations that provide teachers with specific, actionable guidance for implementing these practices in their classrooms.
Project STAIR Algebra Project STAIR - YouTube October |2018 | Project STAIR (smu.edu) Project STAIR: Supporting Teaching of Algebra: Individual Readiness | Simmons School of Education & Human Development | SMU	A U.S. Department of Education Office of Special Education Programs model demonstration project on a system of instructional practices for supporting the algebra-readiness of middle school students with specific learning disabilities in mathematics resulting in videos for teachers to learn instructional strategies for algebra readiness for students with IEPs or in intensive intervention.
Principles for the Design of Mathematics Curricula: Promoting Language and Content Development	Written with the goal of providing guidance to mathematics teachers for recognizing and supporting students' language development processes in the context of mathematical sense-making, this document describes a framework of **Mathematical Language Routines** (MLRs) and includes supporting examples. The MLRs are designed to help teachers address the specialized academic language demands in mathematics when planning and delivering lessons.
UnboundEd Middle School Math Fluency Guide	*The UnboundEd Middle School Math Fluency Guide* offers strategies and rationale for busy teachers who want to develop procedural fluency in Middle School mathematics. Connect and build on your students' conceptual understanding through structured practice that will help students make sense and expertise with procedures.
TNTP Math Assignment Review Protocol	Use this content-specific protocol to assess the quality of assignments that students are asked to complete. Are they aligned to grade-level standards? Do they give students the opportunity to engage with the mathematical practices? Do they make meaningful connections to the real world?

Observation Tools

Resource	Description
30-Minute Atlas Protocol	**Protocol** describing a collaborative process for examining students' performance data to inform next steps in teaching.
RESET Explicit Instruction Rubrics	**Rubric, webinar, manual, and related resources** focused on explicit instruction. The Recognizing Effective Special Education Teachers (RESET) project, funded by U.S. Department of Education Institute for Education Sciences (IES) and led by Evelyn Johnson at Boise State University, developed a series of rubrics based on evidence-based practices for students with high-incidence disabilities. One set of rubrics focuses on explicit instruction. Based on the main ideas of Explicit Instruction, the Explicit Instruction Rubric was designed for use by supervisors and administrators to reliably evaluate explicit instructional practice, to provide specific, accurate, and actionable feedback to special education teachers about the quality of their explicit instruction, and ultimately, improve the outcomes for students with disabilities.
Achieve the Core's Instructional Practice Guide	The Instructional Practice Guide (IPG) is a K–12 classroom observation rubric that prioritizes what is observable in and expected of classroom instruction when instructional content is aligned to college- and career-ready (CCR) standards. It purposefully focuses on the limited number of classroom practices tied most closely to content of the lesson.
"What to Look For" Observation Guides	Created by the Massachusetts Department of Education, these guides briefly outline what content and practices should be observed during instruction for a specific grade or course.

Evidence of Learning Tools

Resource	Description
Student Work Analysis Protocol	**Protocol** describing a process that groups of educators can use to discuss and analyze student work. It is intended to be applicable across subjects and grades, including literacy, mathematics, science, the arts, and others. Analyzing student work gives educators information about students' understanding

Resource	Description
	of concepts and skills and can help them make instructional decisions for improving student learning.
Instructional Rounds / Atlas Protocol	**Protocol** describing a process for conducting 8-minute instructional rounds in groups.
Calibration Protocol for Scoring Student Work	**Protocol** describing a process that groups of educators can use to discuss student work in order to reach consensus about how to score it based on rubric/scoring criteria. It is intended to be applicable across subjects and grades, including literacy, mathematics, science, the arts, and others. Examples of student work that can be used as practice for calibration are included as appendices.

Additional Tools and Resources

Resource	Description
School Reform Initiative (SRI)	**Website** with a wide range of protocols that support teaching and learning. The mission of the School Reform Initiative is to create transformational learning communities that are fiercely committed to educational equity and excellence.
National School Reform Faculty (NSRF)	**Website** with a wide range of protocols that can be used in collaborative settings, such as PLCs and Critical Friends groups, to enhance teaching and learning.
Sample Teaching Activities to Support Core Competencies of SEL	**Document** drawing on CASEL reviewed evidence-based programs to identify and describe some of the most common strategies used to promote student SEL.
Using Explicit and Systematic Instruction to Support Working Memory	**Article** with implementation examples in elementary expository text and mathematics lessons
Effective Practices Alignment Matrix	**Tool** describing Montana's Effective Practices Alignment Matrix of Three major national and statewide professional development initiatives: the Danielson Framework, Teaching Works High-leverage Practices (HLPs), and the Council for Exceptional Children HLPs for Students with Disabilities — using the effective practices ratings system developed by John C. Hattie.

Resource	Description
Collaborative Team Tool Kit	**Toolkit** from the State of New Jersey's Collaborative Teams intended to help schools establish productive collaborative teams of teachers and administrators working and learning together to help their students.
Questioning strategies to engage all learners	**Guide** to questioning strategies for teachers. Teachers strategically vary the types of questions they ask to generate meaningful dialog that supports the development of higher-order thinking skills.
Strategic Questioning	**Article** on strategic questioning. Strategic questioning is intentional, systematic and targets students' learning. Within such a process, students are not just listening and answering questions, but they are also involved in analyzing their teacher and peer's questions, raising more questions, taking turns to discuss each other's answers, and evaluating them.
Student Discourse	**Article** on six ways to move students' thinking to deeper understanding.
Coherence Map (achievethecore.org)	Standards relate to one another, both within and across grades. The Coherence Map illustrates the coherent structure that is fundamental to college- and career-ready standards. It can be a useful tool when planning instruction to close gaps in student understanding.
Videos | Online Resources (corwin.com) Reproducibles | Online Resources (corwin.com) Video 2.4. Continual Assessment for Daily Planning Video 3.1. What We Mean by Tasks With Rigor Video 3.2. Questioning That Guides Learning Video 3.3. Student Discourse That Builds Understanding Video 5.2. Student Collaboration and Discourse for Deep Learning Video 5.3. Grouping Strategies for Deep Learning Video 7.2. Feedback That Fosters Learning Video 7.3. Feedback That Fosters Perseverance Video 7.4. Growth Mindset: The Students' Perspective	"In these videos, you'll meet teachers and hear their perspectives about teaching mathematics. You'll also visit their classrooms. In addition, we include effect sizes in the margins for easy reference. We also include a number of teaching tips and definitions of terminology in the margins as well as a number of reproducible forms and tools that you can use in your classroom." Hattie, J., Fisher, D., Frey, N., Gojak, L.M., Moore, S. D., & Mellman, W. (2016). *Visible learning for mathematics, grades K-12: What works best to optimize student learning*. Corwin Press.

Resource	Description
UDL Guidelines in Math - Google Docs SF USD UDL for Math	Guidelines on providing universal accessibility supports when teaching mathematics
RI Math Project UDL, Differentiation, and Scaffolding Module	RI Math Project learning module with videos, materials, and implementation resources on UDL, Differentiation, and Scaffolding to ensure core access and rigor in elementary and middle school now embedded in a professional learning course as well as grouped in a Google site
RI Math Project Effective Math Core Instruction Modules Part 1 and 2	RI Math Project created two modules on Effective Core Instruction. Part 2 has specific teacher guidance on ensuring all students participate and reducing bias. Module resources include video, slide deck, handouts, implementation resources, student and teacher look-fors, readings.
RI Math Project Supporting Language Development in Mathematics	Learn about the language of mathematics and the importance of vocabulary. Understand the challenges of word problems. Discover instructional practices to support language and vocabulary development.
Evidence-based one-pagers of mathematics instruction from the RI Math Project to support teachers of students with LD, ADHD, or unfinished learning in specific areas: fluency; CRE; cover-copy-compare; additive compare, change, combine, multiplicative equal groups and comparison	A set of concise handouts with explicit how-to instruction for teachers to implement specific mathematics strategies with students who need more explicit instruction to make progress in the core. Each handout includes steps, videos, rationale, and target student population.
RI Math Project Features of Assessment for Math within an MTSS	Module resources: pre-reflection, video, slide deck, handouts, implementation resources, assessment resources
High-Quality Math Instruction IRIS Center Module Page 4: Explicit, Systematic Instruction Page 5: Visual Representations Page 6: Schema Instruction Page 7: Metacognitive Strategies Page 8: Effective Classroom Practices IRIS | Perspectives & Resources (vanderbilt.edu)	Subsection *What evidence-based mathematics practices can teachers employ?* • Describe some evidence-based practices for teaching mathematics • Recognize effective classroom practices that promote and support the implementation of high-quality mathematics instruction **Note**: The Explicit Systematic Instruction video employs a mnemonic to support initial instruction that focused on developing *conceptual understanding* of the trigonometric ratios.
Math Interventions for Students With Autism Spectrum Disorder: A Best-	An overview of mathematics interventions for children and adolescents with ASD

Resource	Description
Evidence Synthesis - Seth A. King, Christopher J. Lemons, Kimberly A. Davidson, 2016 (sagepub.com)	Included studies: • met the design standards of the What Works Clearinghouse (2014) • showed moderate to large effect sizes • focused on functional and computational skills for students with a comorbid diagnosis of intellectual disability (ID)
Checklist to Implement and Plan the Worked Solutions Strategy	This document provides an implementation checklist to scaffold algebra tasks for students with LD using worked solutions. While the focus of the checklist and referenced article is on students with LD, the approach is also useful when working with any student with unfinished learning.
Examples of Social and Emotional Learning in Elementary Mathematics Instruction	CASEL document produced in collaboration with MA DESE which identifies and describes some of the most common strategies used to promote student SEL in mathematics based upon CASEL reviews of evidence-based programs
IRIS Center Math Videos	Math Instruction Video Playlist on Metacognitive Strategies and Explicit, Systematic Instruction The IRIS Center is a national center dedicated to improving education outcomes for all children, especially DAS, through the use of effective evidence-based practices and interventions. **Note:** The Explicit Systematic Instruction video employs a mnemonic to support initial instruction that focused on developing *conceptual understanding* of the trigonometric ratios. The high school Metacognitive Strategies video demonstrates how the mnemonic supports a student in applying her already developed *understanding* of the trigonometric ratios when solving a word problem.

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Section 4: High-Quality Learning through Assessment

Introduction and Overview

As described in previous sections, the curriculum frameworks are built upon the foundation of rigorous standards and high-quality curriculum materials (HQCMs). Section 3 discussed how this foundation informs high-quality instruction. This section focuses on how it should also ensure high-quality learning through assessment. When properly designed and implemented, a comprehensive assessment system provides multiple perspectives and sources of data to help educators understand the full range of student achievement. Assessment information may be used to evaluate educational programs and practices and make informed decisions related to curriculum, instruction, intervention, professional learning, and the allocation of resources to better meet students' needs.

Assessment information also informs educators and families on student performance and their relationship to ongoing instructional practice. Various types of assessments are required because they provide different types of information regarding performance. A comprehensive assessment system must be appropriate for the student population and address the assessment needs of students at all grade levels, including those who speak languages other than English, are differently-abled, who struggle, or who excel. Most multilingual learners and differently-abled students participate in typical statewide and classroom-based assessment systems for mathematics.

Student learning is most maximized with an aligned system of standards, curriculum, instruction, and assessment. When assessment is aligned with instruction, both students and teachers benefit. Students are more likely to learn because instruction is focused and because they are assessed on what they are taught. Teachers are also able to focus, making the best use of their time. Assessments are only useful if they provide information that is used to support and improve student learning.

Assessment inspires us to ask these hard questions:

- "Are we teaching what we think we are teaching?"
- "Are students learning what we want them to learn?"
- "Is there a way to teach the subject and student better, thereby promoting better learning?"

Section 4 will orient you to the purposes and types of assessment, the concepts of *validity*, *reliability*, and *fairness* in assessment, factors to consider when selecting or developing assessments, and considerations when assessing differently-abled students or multilingual learners.

Purposes and Types of Assessment

Assessment has an important and varied role in public education. Assessments are used to inform parents about their children's progress and overall achievement. Teachers use assessment to make decisions about instruction, assign grades, and determine eligibility for special services and program placement. They are used by evaluators to measure program and instructional effectiveness. They are also used to track progress toward school and LEA goals set by the state in accordance with federal regulations. When it comes to assessment of student learning, the why should precede the how because assessments should be designed and administered with the purpose in mind. The vast majority of assessments are used for one of three general purposes: to inform and improve instruction, to screen/identify (for interventions), and to measure outcomes.

When assessments are used to inform instruction, the data typically remain internal to the classroom. They are used to provide specific and ongoing information on a student's progress, strengths, and weaknesses, which can be used by teachers to plan and/or differentiate daily instruction. This daily process is most typically referred to as formative assessment. However, interim and summative assessments can also be used to impact instructional decision-making, though not in the short-cycle timeline that characterizes formative assessments. Assessments such as unit tests and even state assessment data can be used to reflect on and inform future instructional decisions.

When assessments are used to screen/identify, the data also typically remain internal to the school or LEA. Assessments that are used primarily to screen are administered to the total population of students and generally assess key skills that are indicators of students' larger skill set, rather than an in-depth analysis of the standards. They should be relatively quick to administer and easy to score. Assessments used for screening purposes can inform decisions about the placement of groups of students within an academic program structure or individual students' needs for academic interventions or special programs. When needed, screening assessments are followed by diagnostic assessments to determine if more targeted intervention is necessary or if a student has a disability.

Finally, when assessments are used to measure outcomes, data are communicated to parties external to the classroom. Whether it is a unit test that is entered into a grade book and communicated to parents or a standardized test that is reported to the State. Assessments used to measure outcomes attempt to measure what has been learned so that it can be quantified and reported. No single type of assessment, and certainly no single assessment, can serve all purposes.

From informal questioning to final exams, there are countless ways teachers may determine what students know, understand, and are able to do. The instruction cycle generally follows a pattern of determining where students are with respect to the standards being taught before instruction begins, monitoring their progress as the instruction unfolds, and then determining what knowledge and skills are learned as a result of instruction. Assessments, based on when they are administered relative to instruction, can be categorized as formative, summative, or interim.

The primary purpose of **formative assessment** is to inform instruction. As an instructional practice, it is described more fully in Section 3 of this framework. The Chief Council of State School Officers (CCSSO, 2018) updated its definition of formative assessment in 2021 and defines formative assessment in the following way:

Formative assessment is a planned, ongoing process used by all students and teachers during learning and teaching to elicit and use evidence of student learning to improve student understanding of intended disciplinary learning outcomes and support students to become self-directed learners.

Effective use of the formative assessment process requires students and teachers to integrate and embed the following practices in a collaborative and respectful classroom environment:

- *Clarifying learning goals and success criteria within a broader progression of learning;*
- *Eliciting and analyzing evidence of student thinking;*
- *Engaging in self-assessment and peer feedback;*
- *Providing actionable feedback; and*
- *Using evidence and feedback to move learning forward by adjusting learning strategies, goals, or next instructional steps.*

Additionally, formative assessment is integrated throughout instruction with the purpose of gathering evidence to adjust teaching, often in real time, to address student needs (Black and Wiliam, 2010), and capitalize on student strengths. There is ample evidence to support that this process produces "significant and often substantial learning gains" (Black and Wiliam, 2010) and these gains are often most pronounced for low-achieving students. Eliciting evidence of student thinking as part of the formative assessment process should take varied forms. Examples of strategies for gathering evidence of learning during the formative assessment process include exit slips, student checklists, one-sentence summaries, misconception checks (Alber, 2014), targeted questioning sequence, conferences, and observations.

Formative assessment becomes particularly powerful when it involves a component that allows for student self-assessment. When teachers clearly articulate learning goals, provide criteria for proficiency in meeting those goals, and orchestrate a classroom dialogue that unveils student understandings, students are then positioned to monitor their own learning. This self-knowledge, coupled with teacher support based on formative assessment data, can result in substantive learning gains (Black and William, 2010). Learner involvement in monitoring progress on their goals strengthens engagement for all students but is especially important for differently-abled students. Specific feedback comparing the students' achievement against the standard — rather than only against other students — increases personal performance. With specific feedback, learners should then have the opportunity to resubmit some items in response. Opportunities for students to monitor their own progress and make improvements based on specific feedback connect to the Social Emotional Learning competency of **Self-management** — learning to manage and express emotions appropriately, controlling impulses, overcoming challenges, setting goals, and persevering and Self-awareness Learning Standards 1B — I can identify when help is needed and who can provide it. **Self-Awareness** means students understand their areas of strength as well as areas of need. This skill is strengthened as they monitor their progress. By incorporating Universal Design for Learning guidelines, assessment feedback that is relevant, constructive, accessible, specific, and timely with a focus on moving the learner toward mastery is more productive in promoting engagement. The assessment process creates a continuous feedback loop, which systematically checks for progress and identifies strengths and weaknesses to improve learning gains during instruction.

Summative assessments are formal assessments that are given after a substantial block of instructional time, for example at the end of a unit, term course, or academic year. **Interim assessments** are administered during instruction and depending on the type of interim assessment can be used to screen students, inform instruction, or measure outcomes. By design and purpose, high-quality summative and interim assessments are less nimble in responding to student strengths and needs than formative assessments. They provide an overall picture of achievement and can be useful in predicting student outcomes/supports or evaluating the need for pedagogical or programmatic changes. These assessments should be written to include a variety of item types (e.g., selected response, constructed response, extended response, performance tasks) and represent the full scale of Webb's Depth of Knowledge (DOK). To maximize the potential for gathering concrete evidence of student learning as facilitated by curriculum and instruction, educators should routinely draw upon the assessments provided within their HQCMs (RIDE, 2012).

State assessments are summative assessments that are given annually and provide a valuable "snapshot" to educators and families and help us see how we are doing compared with other districts, compared with the state as a whole, and compared against several other high-performing states. State assessments only account for about 1 percent of most student's instructional time. Results from state assessments that are part of a comprehensive assessment system keep families and the public at large informed about school, district, and state achievement and progress.

Interim assessments include screeners and diagnostic assessments. Screening assessments are a type of interim assessment used as a first alert or indication of specific instructional need and are typically quick and easy to administer to a large number of students and easy to score. Assessments used for screening purposes can inform curriculum decisions about instruction for groups of students and for individual student's academic supports. Schools and districts often use interim assessments to screen and monitor student progress across the school year.

Examples of these assessments used in schools and districts include STAR, i-Ready, NWEA, IXL, and aimsweb. Some of these screening tools also have progress monitoring capability to track a student's response to intervention at a more frequent interval. Progress monitoring tools may be general outcome measures or mastery measures. While general outcome measures (GOMs) measure global skill automaticity, mastery measurement closely looks at one aspect or specific skill. When needed, screening assessments can be followed by more intensive diagnostic assessments to determine if targeted interventions are necessary. Diagnostic assessments are often individually administered to students who have been identified through the screening process. The diagnostic assessments help to provide greater detail of the student's knowledge and skill.

Progress Monitoring	
General Outcome Measures (GOM)	GOMs measure automaticity of basic skills in reading, math, spelling and written expression as well as monitor readiness skills in literacy and numeracy. While GOMs do not measure all aspects of reading or math, they do serve as a predictive indicator of academic competence in these fundamental content areas and are typically used for setting intervention goals.
Mastery Measures	Mastery measures determine how much a student already knows about and where instruction should begin as well as determining when a student has mastered a particular skill taught. They help determine if the student is learning the specific skills as a result of an intervention and help identify where and how to intervene.
Progress Monitoring Tools Chart	This chart includes measures designed to assess progress towards end-year goal (e.g., oral reading fluency) and measures designed to assess mastery towards short-term skills (e.g., letter naming fluency). The chart reviews the peer-reviewed research on progress monitoring tools submitted by the vendors and reports on reliability, validity, bias analysis, sensitivity for reliability and validity of slope, alternate forms, decision rules, administration format, scoring time, scoring format, ROI and EOY benchmarks for each measure. Click on the tabs and tools names to see additional information including detailed data.
IRIS Center Information Brief	This brief describes and compares two types of progress monitoring, Mastery Measures and General Outcome Measures, providing math and ELA examples and characteristics of each measure.

Performance assessments/tasks can be an effective way to assess students' learning of the standards within a high-quality curriculum. Performance assessments/tasks require students to apply understanding to complete a demonstration performance or product that can be judged on performance criteria (RIDE, 2012). Performance assessments can be designed to be formative, interim, or summative assessments of learning. They also allow for richer and more authentic assessment of learning. Educators can integrate performance assessments into instruction to provide additional learning experiences for students. Performance tasks are often included as one type of assessment in portfolios and exhibitions, such as those used as part of Rhode Island's Proficiency Based Graduation Requirements.

	Inform Instruction	Screen/Identify	Measure Outcomes
Summative	Generally, not used as the primary source of data to inform instruction. May be useful in examining program effectiveness.	Generally, not used as the primary source of data to screen/identify students. May be one of multiple sources used.	Primary purpose is to measure outcomes (at classroom, school, LEA, or state level). Can be used for accountability, school improvement planning, evaluation, and research.
Formative	Primary purpose is to inform instruction.	Generally not used to screen/identify students.	Generally not used to measure long term outcomes; rather, it is used to measure whether students learned what was just taught before moving on to instructional "next steps". Evidence gathered as part of the formative assessment process may inform a referral to special education and may be used to help measure short-term objectives on IEPs.
Interim	May be used to inform instruction.	May be used to screen/identify students.	May be used to measure outcomes in a longer instructional sequence (e.g., end of a unit of study or quarter, semester, MTSS intervention goal, IEP goal). May be part of a special education referral.

The authors of *Principles to Actions: Ensuring Mathematical Success for All* (2014) succinctly summarize the current thinking in mathematics assessment in this statement: "An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency, with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement." In other words, good assessment in mathematics is not about assigning a grade, but rather about gathering and using evidence to maximize student learning. Assessment in mathematics is not "done to" students, but is "done for "students.

These principles of good assessment are reflected in assessments contained within a high-quality mathematics curriculum. As such, educators should strive to make use of the assessments contained within their high-quality curriculum. As part of the high-quality review process, assessments are rated for their alignment to the standards and their ability to provide actionable data to improve instruction. The rubric used to evaluate the quality of assessments within a program highlights the importance of varied assessment strategies and instruments for determining student proficiency as well as accessing prior knowledge, identifying student errors and misconceptions, and opportunities for providing feedback. The rubric also emphasizes the importance of positioning students to self-evaluate and monitor their own learning (EdReports, 2021).

When access to curriculum embedded high-quality assessment is not an option, educators should take critical care in selecting assessments that are directly tied to the standards and their instruction. Such assessments should reflect the required rigor of the standards assessed, make connections to the relevant mathematical practices, be designed to provide equitable access for all students, and serve the ultimate goal of providing data to improve the teaching and learning of mathematics (NCTM, 2014).

What do educators need to know about validity, reliability and fairness?

Assessments must be designed and implemented to accurately collect student information. To do this they should all possess an optimal degree of

- *Validity* (the degree to which the assessment measures what it is supposed to measure — i.e., what is defined by the standards),
- *Reliability* (the consistency with which an assessment provides a picture of what a student knows and is able to do), and
- *Fairness* (lacks bias, is accessible, and is administered with equity) (RIDE, 2012).

In other words, within an assessment, the items must measure the standards or content. It is also critical that the assessment provide information that demonstrates an accurate reflection of student learning. Ensuring fairness is equally important within the assessment, particularly for differently-abled and multilingual learners, because lack of accessibility can impact validity. For example, an assessment may not measure what it was designed to measure if students cannot access the assessment items or stimuli due to linguistic barriers or inattention to other demonstrated learning needs.

One component of ensuring fairness is using assessments that are accessible to all students. Accessible assessment practices may include offering assessments in different modalities (e.g., Braille, oral) or languages, allowing students to respond in different modalities, or providing additional accommodations for students. Accessibility features are available for all students to ensure universal access to the assessment. To further support differently-abled students and multilingual learners, accommodations are also available on all state assessments. Accommodations refer to changes in setting, timing (including scheduling), presentation format, or response format that do not alter in any significant way what the test measures, or the comparability of the results. For example, reading a test aloud may be appropriate when a student is taking a history assessment, but would not be appropriate to assess a student's decoding ability. When used properly, accessibility features and appropriate test accommodations remove barriers to participation in the assessment and provide students with diverse learning needs an equitable opportunity to demonstrate their knowledge and skills.

To ensure language access for MLLs, universal accessibility features and accommodations can be leveraged during administration of assessments, in a manner consistent with Rhode Island State Assessment Program policy. For example, breaks and familiar test administrators are available to MLLs on all statewide assessments except Pre-SAT/SAT. For additional information about accessibility features, please see RIDE's Accommodations and Accessibility Features Manual. Accommodations are also available to MLLs on all statewide assessments. Examples of accommodations include bilingual dictionaries, reading aloud the test directions in the student's native language, and Spanish editions of math and science assessments. A full list of accommodations available to MLLs on each state assessment is available in RIDE's Accommodations and Accessibility Features Manual.

For both MLLs and DAS, assessment accommodations should reflect instructional accommodations used on a regular basis with a student. Educators evaluate the effectiveness of accommodations through data collection and the consideration of the following questions:

1. Did the student use the accommodation consistently?
2. Did the accommodation allow the student to access or demonstrate learning as well as their peers?
3. Did the accommodation allow the student to feel like a member of the class?
4. Did the student like using the accommodation?

Most students with IEPs participate in regular statewide assessments with accommodations as outlined in the IEP. DAS who receive testing accommodations must take the same statewide assessment as peers without IEPs. IEP team members collaborate to select accommodations based on educational needs demonstrated by current data, not based on placement or disability category. All students with disabilities should be included in educational accountability systems and a small percentage (~1%) of students with significant cognitive impairments participate in alternate state assessment. Educators should engage students and families in decisions about appropriate testing accommodations or participation in alternate assessments (i.e., DLM and Alternate ACCESS).

IDEA also speaks to accommodations on district assessments as well as statewide assessments. According to IDEA Sec. 300.320(a)(6), each child's IEP must include a statement of any individual appropriate accommodations that are necessary to measure the academic achievement and functional performance of the child on state **and districtwide assessments** consistent with section 612(a)(16) of the Act. When determining accommodations for district assessments, IEP teams, including the general educator, must consider the difference between target skills (the knowledge or skills being assessed) and access skills (needed to complete the assessment, but not specifically being measured) along with data on the strengths and needs of the individual student.

Another component for ensuring fairness is making sure the items do not include any bias in content or language that may disadvantage some students. For example, when assessing multilingual learners, it is important to use vocabulary that is widely accessible to students and avoid colloquial and idiomatic expressions and/or words with multiple meanings when it is not pertinent to what you are measuring. Whenever possible, use familiar contexts or objects like classroom or school experiences rather than ones that are outside of school that may or may not be familiar to all students. Keep sentence structures as simple as is possible while expressing the intended meaning.

Even with valid, reliable, and fair assessments, it is important for educators to consider multiple data points to ensure that they have a comprehensive understanding of student strengths and needs, especially when supporting DAS and MLLs. In addition to interim and diagnostic assessment, sources of information can range from observations, work samples, and curriculum-based measurement to functional behavioral assessments and parent input. These data points should be gathered within the core curriculum by general educators, rather than only by those providing specialized services, because data should guide daily decisions about instruction within general education. Multiple sources of information help educators collaborate to develop a comprehensive learner profile of strengths and needs. Educators can analyze the learning environment against that profile to identify necessary scaffolds and accommodations to remove barriers for DAS. Multiple sources of data are also important, seeing as language access can impact student data from content assessments in English.

Selecting and Developing Assessments

Building or refining a comprehensive assessment system begins by agreeing upon the purposes of the assessments the LEA will administer. One assessment cannot answer every question about student learning. Each type of assessment has a role in a comprehensive assessment system. The goal is not to have some — or enough — of each type; rather it is to understand that each type of assessment has a purpose and, when used effectively, can provide important information to further student learning. Some questions educator teams may ask themselves as part of any discussion of purpose include:

- “What do we want to know about student learning of the standards?”
- “What do we want to learn about students’ skills and knowledge?”
- “What data do we need to answer those questions?”

Once claims and needs are identified, the appropriate assessments are selected to fulfill those data needs by asking: “Which assessment best serves our purpose?” For example, if a teacher wants to know if students learned the material just taught and identify where they may be struggling to adjust the next day's instruction, the teacher may give a short quiz which asks students a few questions targeting a specific skill. Whereas, if the teacher wanted to know if the students were proficient with the content taught during the first semester, the teacher may ask students to complete a longer test or performance task where students apply their new learning, thus measuring multiple standards/skills.

In addition to considering what purpose an assessment will serve, attention must be paid to the alignment of the assessment with the curriculum being used by the LEA. Curriculum materials embed assessments as part of the package provided to educators. In turn, educators must consider whether the assessments included meet the breadth of purposes and types needed for an assessment system that informs instruction and provides information about student learning. A good starting place is to review what assessments are available within the high-quality instructional materials, identify gaps and weaknesses, and develop a plan for which additional assessments may need to be purchased or developed. Remember any review of assessments needed involves a close use of the standards and universal design guidelines. Providing options in the way assessments are represented and allowing for students to demonstrate their understanding through multiple means of action and expression benefits all students, especially MLLs and DAS.

Assessments that are not adequately aligned with the LEA’s adopted curriculum and universal design are not accurate indicators of student learning. This is especially important when assessment data are used in high-stakes decision-making, such as student promotion or graduation. Because every assessment has its limitations, it is preferable to use data from multiple assessments and types of assessments. By collecting data from multiple sources, one can feel more confident in inferences drawn from such data. When curriculum, instruction, and assessment are carefully aligned and working together, student learning is maximized.

Finally, when developing or selecting assessments, knowing whether an assessment is a good fit for your needs requires a basic understanding of item types and assessment methods and their respective features, advantages, and disadvantages. Though this is certainly not an exhaustive list, a few of the most common item types and assessment methods include selected response, constructed response, performance tasks, and observations/interviews. See *Comprehensive*

Assessment System: Rhode Island Criteria and Guidance (2012) for a discussion of the advantages and disadvantages of each method.

Assessment Considerations for MLLs and DAS

In addition to selecting and designing appropriate assessments, it is critical that educators use sound assessment practices to support MLLs and DAS during core instruction. Assessments offers valuable insight into MLL and DAS learning, and educators should use this data to plan and implement high-quality instruction. Through formative assessment, educators of mathematics play a central role in providing feedback to MLLs on content and disciplinary language development and DAS on progress towards IEP goals.

As with academic content, a comprehensive assessment system is essential for monitoring the language development of MLLs. To assess English language proficiency, RIDE has adopted ACCESS for ELs as its statewide summative assessment. However, students cannot acquire a second language in a single block of the school day. Thus, it is imperative that educators and administrators develop systems for conducting ongoing formative assessments of content driven language instruction. This approach aligns to WIDA ELD Standards Framework as well as the Blueprint for MLL Success, both of which explicitly call for disciplinary language teaching within the core content areas.

The same integration of evidence-based assessment practices for DAS is needed within the general education curriculum. Seventy percent of RI students with IEPs are in general education settings at least 80% of their day. IEP goals are meant to measure and improve student progress *within* the general education curriculum. The specially-designed instruction is typically not happening separately, but in connection with the classroom instruction and curriculum. The general educator and special educator work in consultation to use classroom data to measure progress on an IEP goal along with any additional measures indicated in the IEP.

DAS may benefit from data-based individualization (DBI) to improve their progress in the general education curriculum. DBI is an iterative, problem-solving process that involves the analysis of progress-monitoring and diagnostic assessment data. Diagnostic data from tools such as standardized measures, error analysis of progress-monitoring data and work samples, or functional behavioral assessments (FBA) are collected and analyzed to identify the specific skill deficits that need to be targeted. The results of the diagnostic assessment, in combination with the teacher's analysis of what features of instruction need to be adjusted to better support the student, help staff determine how to individualize the student's instructional program to meet the individual student's unique needs and promote progress in the general education curriculum. The diagnostic process allows teachers to identify a student's specific area(s) of difficulty when lack of progress is evident and can inform decisions about how to adapt the intervention (National Center on Intensive Intervention, 2013).

Assessment to Support MLLs in High-Quality Core Instruction

The 2020 Edition of the WIDA ELD Standards Framework is different from previous iterations in that it contains **proficiency level descriptors** by grade level cluster to support developmentally appropriate, content-driven language learning. In addition to assessing MLLs' content learning in their home languages when possible for added validity, educators of mathematics should draw on the WIDA proficiency level descriptors to design or amplify formative assessments tracking MLLs' language development in mathematics.

As with the formative assessment process in academic content, establishing clear learning goals is the first step in improving student understanding of intended content-based language outcomes. To use the proficiency level descriptors, educators must determine **the mode of communication** (i.e., whether they are assessing interpretative or expressive language) and select the corresponding set of descriptors. This determination will likely be made when the educator identifies the language goals. **Expressive language** refers to speaking, writing, and representing, whereas **interpretative language** includes listening, reading, and viewing.

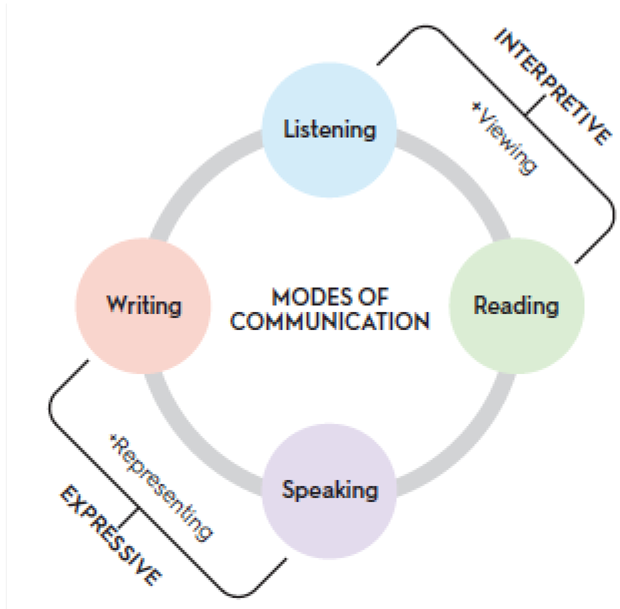

Image Source: [2020 Edition of WIDA ELD Standards Framework]

The proficiency level descriptors should serve as a key resource to educators when refining language goals for assessment purposes, as the proficiency level descriptors highlight characteristics of language proficiency at each level. These descriptors are organized according to their discourse, sentence, and word dimensions. At the discourse level, as shown in the following table, the 2020 Edition distinguishes between language features that contribute to organization, cohesion, or density.

Grades 4-5 WIDA Proficiency Level Descriptors for the Expressive Communication Mode (Speaking, Writing, and Representing)
Toward the end of each proficiency level, when scaffolded appropriately, multilingual learners will...

Criteria	End of Level 1	End of Level 2	End of Level 3	End of Level 4	End of Level 5	Level 6
	Create coherent texts (spoken, written, multimodal) using...					
DISCOURSE Organization of language	short sentences linked by topic to convey an emerging sense of purpose (to inform, explain, argue, narrate)	sentences that convey intended purpose with emerging organization (topic sentence, supporting details)	short text that conveys intended purpose using predictable organizational patterns (signaled with some paragraph openers: *first, and then, then*)	expanding text that conveys intended purpose using generic (not genre-specific) organizational patterns across paragraphs (introduction, body, conclusion) with a variety of paragraph openers	text that conveys intended purpose using genre-specific organizational patterns (statement of position, arguments, call to action)	text that conveys intended purpose using genre-specific organizational patterns with strategic ways of signaling relationships between paragraphs and throughout text *(the first reason, the second reason, the evidence is...)*
	Connect ideas across a whole text through...					
DISCOURSE Cohesion of language	some frequently used cohesive devices (repetition, demonstratives)	some formulaic cohesive devices (pronoun referencing, etc.)	a growing number of cohesive devices (emerging use of articles to refer to the same word, synonyms, antonyms)	an expanding variety of cohesive devices (given/new, whole/part, class/subclass)	a flexible number of cohesive devices (substitution, ellipsis, given/new)	a wide variety of cohesive devices used in genre- and discipline-specific ways
	Elaborate or condense ideas through...					
DISCOURSE Density of language	a few types of elaboration (adding familiar adjectives to describe nouns: *maple syrup*)	some types of elaboration (adding newly learned or multiple adjectives to nouns *(thick, sweet, sticky maple syrup)*	a growing number of types of elaboration (adding articles or demonstratives to nouns: *the dark syrup*)	a variety of types of elaboration (adding in a variety of adjectives including concrete and abstract nouns: *the long, slow process...*)	a wide variety of types of elaboration (adding in embedded clauses after the noun: *the sap which boiled for six hours...*)	flexible range of types of elaboration that includes embedded clauses and condensed noun groups (elaborating: *a sweet sap that turned into a delicious syrup after hours of boiling* and condensing through nominalization: *this tedious process*)

Image Source: [2020 Edition of WIDA ELD Standards Framework]

During formative assessments, educators will not likely draw on all dimensions of language at once for assessment purposes. For instance, an exit ticket that asks students to produce two to three sentences would not be an appropriate language sample for assessing progress on organization of language. To adequately assess this discourse-level dimension of language, students would need authentic opportunities to demonstrate proficiency. An assessment item that calls for less than one paragraph or extended oral remarks, therefore, may not suffice for this purpose.

Rather than creating separate assessments to monitor progress towards disciplinary language development, educators should aim to augment assessments that are already part of their local core curricula. For example, multiple modalities could be incorporated into existing content assessments, allowing students to orally explain how they arrived at a particular solution or claim. This practice of amplifying existing materials with additional modalities aligns with UDL guidelines by providing multiple means of representation (perception, language, and symbols) and multiple means for students to demonstrate their understanding (physical action, expression, and communication) — a critical design element for MLLs who need daily explicit speaking, listening, reading, and writing instruction.

Assessment to Support Differently-Abled Students in High-Quality Core Instruction

Differently-abled students are best supported when general and special educators use Universal Design for Learning to collaboratively design and plan assessments aligned to clear learning goals to ensure they measure the intended goals of the learning experience. Flexibility in assessment options will support learners in demonstrating their knowledge. All learners can benefit from practice assessments, review guides, flexible timing, assistive technologies, or support resources and help reduce the barriers that do not change the learning goals being measured. In addition to improving access, flexible assessment options may decrease perceived threats or distractions so that learners can demonstrate their skills and knowledge. For example, a student with specific support needs for fine motor skills may be more able to participate in demonstrating knowledge of how to make a square when given the opportunity to drag and drop line segments in a technology tool rather than use a pencil on paper or a marker on a white board.

Educators can use high-leverage practices (HLPs) to leverage student learning across the content areas, grade levels, and various learner abilities. The HLPs contain specific evidence-based practices in four domains: Instruction, Assessment, Collaboration, and SEL.

High-leverage practice #6, on the use of student assessment data to analyze instructional practices and make necessary adjustments that improve student outcomes, highlights the importance of ongoing collaboration between general education and special education in this practice (McLeskey, J, 2017). Information from functional skills assessments, such as those provided by an occupational therapist or speech language therapist, can provide critical information for general educators to use when designing accessible assessments or discussing necessary accommodations to classroom and district assessments. When differently-abled students are not making the level of progress anticipated, the data-based individualization process is a diagnostic method that can help to improve the instructional experience and promote progress in the general education curriculum through a tiered continuum of interventions.

Resource	Description
High Leverage Practices Assessment Overview	Assessment plays a foundational role in special education. Students with disabilities are complex learners who have unique needs that exist alongside their strengths. This overview includes a summary of each HLP for assessment.
High-Leverage Practice (HLP) Leadership Guides from the Council for Exceptional Children	**Leadership Guides** for the following HLPs: #4 Use Multiple Sources of Information to Develop a Comprehensive Understanding of a Student's Strengths and Needs #5 Interpret and Communicate Assessment Information with Stakeholders to Collaboratively Design and Implement Educational Programs #6 Use Student Assessment Data, Analyze Instructional Practices, and Make Necessary Adjustments that Improve Student Outcomes #10 Conduct Functional Behavioral Assessments (FBAs)
Participate in Assessment IEP (promotingprogress.org)	This tip sheet provides information about participation in assessment and accommodations for assessments. It includes a brief summary of federal regulations and tips for implementation.

Resource	Description
Accessibility and Accommodations for General Assessments | FAQ | NCEO	This online FAQ includes common questions and answers with hyperlinks to various resources on accessibility, accommodations, and modifications.
IRIS | Page 3: Instructional Versus Testing Accommodations (vanderbilt.edu)	This online learning resource is a portion of an IRIS module that clarifies different types of accommodations.
DLM Assessments - Assessment - Instruction & Assessment World-Class - Rhode Island Department of Education (RIDE)	These documents and professional development modules, along with other relevant general education curriculum materials, may be used to inform instructional planning and goal-setting for students with significant cognitive impairments.
Differently-abled Multilingual Language Learners/ English Learners with Disabilities (ELSWD) The Role of Individualized Education Program (IEP) Teams and Participation in English Language Proficiency (ELP) Assessments	This document elaborates on federal guidance on the role of Individualized Education Program (IEP) teams and ELSWD participation in English Language Proficiency (ELP) assessments.
CAST | UDL Tips for Assessment	This resource provides quick tips and reflection questions to promote accessible assessment.
UDL: Increase mastery-oriented feedback (cast.org)	This component of the interactive UDL matrix supports educators in understanding the importance of accessible and meaningful feedback to students during the assessment process.
Universal Design of Assessments FAQ	NCEO online resource
Impact | Winter 2018/19 Volume 31, Number 2 | Together We Are Better! Collaborative Teaming to Support Authentic Inclusion of Students with Complex Support Needs Cheryl Jorgensen How-To (umn.edu)	In this article, figure 2 gives examples of using student work on a weekly basis to improve instruction and inclusion and monitor progress for a student with complex and intensive needs. Figures 3-4 provide specific support examples for the math case study.

Formative Assessment Resources

Resource	Description
Why Formative Assessments Matter	Introduction to the importance of formative assessments.

Resource	Description
The Impact of Formative Assessment and Learning Intentions on Student Achievement	Summary of findings on formative assessment and student achievement.
CCSSO Revising the Definition of Formative Assessment	This resource provides an overview of the FAST SCASS's revised definition on formative assessment, originally published in 2006. The revised definition includes an overview of the attributes of effective formative assessment and emphasizes new areas emerging from current research, theory, and practice.
Formative_Assessment_10_Key_Questions.pdf (wi.gov)	Consider using this document as one of a variety of resources to support educators' assessment literacy to build student-teacher relationships that improves student outcomes.
Focusing Formative Assessment on the Needs of English Language Learners	In this paper, we examine how formative assessment can enhance the teaching and learning of ELL students in particular.
Formative_Assessment_for_Students_with_Disabilities.pdf (ccsso.org)	This report provides both special education and general education teachers with an introduction to the knowledge and skills they need to confidently and successfully implement formative assessment for students with disabilities in their classrooms through text and video examples. The strategies described in this paper are not limited to use with differently-abled students and work for all students, including those with unfinished learning.

State Summative Assessment Resources

Resource Links
ACCESS for ELLs Alternate ACCESS for ELLs | WIDA (wisc.edu) DLM Assessments NAEP Assessments NGSA Assessments PSAT & SAT Assessments RICAS Assessments Rhode Island State Assessment Program (ri.gov) IEP Team Guidance on Eligibility for Alternate Assessments

Resource Links
Assessment Accommodations - Assessment - Instruction & Assessment - Rhode Island Department of Education (ri.gov) DLM Assessments - Assessment - Instruction & Assessment World-Class - Rhode Island Department of Education (RIDE)

Additional Resources for a Comprehensive Assessment System

Resource Links
Determining Appropriateness of Assessment: Appendix B EQUIP and Learning Forward Professional Learning Community Modules EQUIP Student Work Analysis Tool, SWAT EQUIP Annotated Student Work Initiative Rhode Island Proficiency Framework • Cross-Curricular • English Language Arts • Mathematics • Social Studies • Science Rhode Island Proficiency Framework: Scoring Criteria • ELA • Mathematics • Science • Social Studies Writing Calibration Writing Standards in Action

Screening

Types of Screening Resources	Description and Resource Links
Literacy/Dyslexia Screening	Universal literacy screening should be administered to all students to determine early risk of future reading difficulties. A preventative approach should be used to ensure student risk is revealed early on when intervention is most effective. If a student scores low on these screeners, additional assessments should be administered to determine a student's potential risk for dyslexia, a neurobiological weakness in phonological and orthographic processing. Screeners should include measures of Rapid Automatic Naming (RAN), phonemic awareness, real and pseudoword

Types of Screening Resources	Description and Resource Links
	reading, as well as vocabulary and syntactic awareness, which have implications on prosody, fluency, and ultimately comprehension. For additional guidance, including screening guidance by grade, please see the Massachusetts Dyslexia Guidelines
Early Childhood Screening	Child Outreach is Rhode Island's universal developmental screening system designed to screen all children ages 3 to 5 annually, prior to kindergarten entry. Developmental screenings sample developmental tasks in a wide range of areas and have been designed to determine whether a child may experience a challenge that will interfere with the acquisition of knowledge or skills. Screening results are often the first step in identifying children who may need further assessment, intervention, and/or services at an early age to promote positive outcomes in kindergarten and beyond. Child Outreach Screening - Early Childhood Special Education - Early Childhood - Instruction & Assessment - Rhode Island Department of Education (ri.gov)
MLL Screening	Screening for MLL identification involves completion of the state-approved Home Language Survey (HLS) and potential administration of a Language Screening Assessment, based on responses to the HLS. The guidance below outlines the state-adopted procedure for identifying English Learners in accordance with statue R.I.G.L.16-54-3 and regulation 200-RICR-20-30-3. Additional information on federal and state requirements for screening MLLs can be found in the assessment and placement section of the MLL Toolkit. Multilingual Learner (MLL) Identification, Screening, Placement and Reclassification (May 2021)
Universal Academic Screening	Through universal academic screening, school teams systematically and regularly analyze schoolwide data to determine the health of core instruction. Current academic performance levels from a screener are one type of academic data teams use to identify strengths and areas of need at a grade level as part of a MTSS. Screening within an MTSS Framework Educator Resources for high quality interim assessments Interim Assessments - Assessment - Instruction & Assessment World-Class - Rhode Island Department of Education (RIDE) Assessment Practices Within a Multi-Tiered System of Supports (ufl.edu) Bailey, T. R., Colpo, A. & Foley, A. (2020). Assessment Practices Within a Multi-Tiered System of Supports (Document No. IC-18). Retrieved from University of Florida, Collaboration for Effective Educator, Development, Accountability, and Reform Center website: http://ceedar.education.ufl.edu/tools/innovationconfigurations/

Diagnostic

Resource	Description
IEP Tip Sheet: Measuring Progress Toward Annual Goals | Progress Center (promotingprogress.org)	Suggestions for what to do and what to avoid when **designing progress-monitoring plans** for differently-abled students plus additional resources to learn more.
Student Progress Monitoring Tool for Data Collection and Graphing (Excel) | National Center on Intensive Intervention	This Excel tool is designed to help educators **collect academic progress-monitoring data** across multiple measures as a part of the data-based individualization (DBI) process. This tool allows educators to store data for multiple students (across multiple measures), graph student progress, and set individualized goals for a student on specific measures.
Progress Center High-Quality Academic IEP Program Goals	Recorded **webinar, resources, and materials** on how to set ambitious goals for students by selecting a valid, reliable progress-monitoring measure, establishing baseline performance, choosing a strategy, and writing a measurable goal.
Student-Level Data-Based Individualization Implementation Checklists (intensiveintervention.org)	Teams can use these **checklists to monitor** implementation of the data-based individualization (DBI) process during initial planning and ongoing review (progress-monitoring) meetings.
Tools to Support Intensive Intervention Data Meetings | National Center on Intensive Intervention (NCII)	NCII has created a **series of tools** to help teams establish **efficient and effective** individual student **data meetings.** In the DBI process, the team is focused on the needs of individual students who are not making progress in their current intervention or special education program.
Data Collection and Analysis for Continuous Improvement	Collection and analysis of progress-monitoring data are necessary for understanding how students are progressing towards their IEP goals. These data, along with other data sources, can support ongoing instructional decision making across the continuum of supports and assist teams in evaluating the effectiveness of IEP implementation. In the Data Collection and Analysis for Continuous Improvement menu are **resources and tools for progress-monitoring math and reading, selecting tools, and keeping an implementation log.**
Toolkit_Student-Progress-Monitoring.pdf (transitionta.org)	The National Technical Assistance Center on Transition (NTACT) **toolkit supports data-driven decision-making** for middle and high school students to connect their academic progress and transition

Resource	Description
	goals — includes 50-plus pages of sample tools. Note the inventory on reading, writing, presenting, and study habits (pp. 48–49), and the small group direction instruction recording sheet (p. 71).
Learning Modules Library: Progress Center and NCII Course: The 5 Steps of Data-Based Individualization	From the Progress Center, educators can build knowledge of the data-based individualization (DBI) process that is used to support a **diagnostic practice** and improve instruction for students with intensive learning needs.

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Appendix

Mathematics Glossary, Tables, and Illustrations

A

Absolute value. The absolute value of a real number is its (non-negative) distance from 0 on a number line.
Addition and subtraction within 5, 10, 20, 100, or 1,000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0–5, 0–10, 0–20, or 0–100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.
Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and –3/4 are additive inverses of one another because
3/4 + (–3/4) = (–3/4) + ¾ = 0.
Algorithm/Standard Algorithm: See Table 6.
Algorithm. A procedure for solving a mathematical problem in a finite number of steps that frequently involves repetition of an operation.
Standard algorithm. A step-by-step approach to calculating, decided by *societal convention*, developed for efficiency. Flexible and fluent use of standard algorithms requires conceptual understanding.
Associative property of addition. See Table 3.
Associative property of multiplication. See Table 3.
Assumption. A fact or statement (as a proposition, axiom, postulate, or notion) taken for granted.
Attribute. A common feature of a set.

B

Benchmark fraction. A common fraction against which other fractions can be measured (e.g., ¼, ½, $^2/3$, $^5/5$).
Binomial Theorem. A theorem that gives the polynomial expansion for any whole-number power of a binomial. For powers greater than or equal to zero.
Bivariate data. Pairs of linked numerical observations. *Example: a list of heights and weights for each player on a football team.*
Box plot. A graphic method that shows the distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

C

Cardinality. An understanding of how numbers are ordered, and how to count accurately, matching a number name to the quantity counted.
Cavalieri's Principle. A method, with formula given below, of finding the volume of any solid for which cross- sections by parallel planes have equal areas. This includes, but is not limited to, cylinders and prisms. Formula: Volume = Bh, where B is the area of a cross-section and h is the height of the solid.
Coefficient. A number or variable which is a factor of a term. For example, x is the coefficient in the expression $x(a + b)$ and 3 is the coefficient in the term $3y$.
Commutative property. See Table 3.
Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).
Complex number. A number that can be written as the sum or difference of a real number and an imaginary number. See Illustration 1.
Complex plane. The coordinate plane used to graph complex numbers.
Compose. To put numbers or geometric figures together strategically and purposefully, typically to simplify calculation or to recognize properties
Composite number. A whole number that has more than two factors.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also: algorithm; computation strategy.*
Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also: computation algorithm.*
Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).
Conjugate. The result of writing the sum of two terms as a difference, or vice versa. For example, the conjugate of $x - 2$ is $x + 2$.
Coordinate plane. A plane in which a point is represented using two coordinates that determine the precise location of the point. In the Cartesian plane, two perpendicular number lines are used to determine the locations of points. In the polar coordinate plane, points are determined by their distance along a ray through that point and the origin, and the angle that ray makes with a pre-determined horizontal axis.
Cosine. A trigonometric function that for an acute angle is the ratio between a leg adjacent to the angle when the angle is considered part of a right triangle and the hypotenuse.
Counting number. A number used in counting objects, i.e. a number from the set 1,2,3,4, 5, . . . See Illustration 1.
Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have eight books and three more books are added to the top, it is not necessary to count the stack all over again; one can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

D

Decimal expansion. Writing a rational number as a decimal.
Decimal fraction. A fraction (as 0.25 = ²⁵⁄100 or 0.025 = ²⁵⁄1000) or mixed number (as 3.025 = 3 ²⁵⁄1000) in which the denominator is a power of ten, usually expressed by the use of the decimal point.
Decimal number. Any real number expressed in base ten notation.
Decompose. To take numbers or geometric figures apart strategically and purposefully, typically to simplify calculation or to recognize properties.
Digit. Digits are the numerals 0-9 found in all numbers. 176 is a 3-digit number featuring the digits 1, 7, and 6.
Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.
Directrix. A parabola is the collection of all points in the plane that are the same distance from a fixed point, called the focus (F), as they are from a fixed line, called the directrix.
Dot plot. *See: line plot.*

E

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. *For example,* 643 = 600 + 40 + 3.
Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.
Exponent. The number that indicates how many times the base is used as a factor, e.g., in $4^3 = 4 \times 4 \times 4 = 64$, the exponent is 3, indicating that 4 is repeated as a factor three times.
Exponential function. A function of the form $y = a \cdot b^x$ where $a > 0$ and either $0 < b < 1$ or $b > 1$. The variables do not have to be x and y. *For example,* $A = 3.2 \cdot (1.02)^t$ *is an exponential function.*

Expression. A mathematical phrase that combines operations, numbers, and/or variables (e.g., $3^2 \div a$).

F

Fibonacci sequence. The sequence of numbers beginning with 1, 1, in which each number that follows is the sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144. . .
First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also: median, third quartile, interquartile range*
Fluency. Using efficient, flexible and accurate methods of computing. See Table 7.
Fraction. A number expressible in the form a/b where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a nonnegative number.) *See also: rational number.*
Function. A mathematical relation for which each element of the domain corresponds to exactly one element of the range.
Function notation. A notation that describes a function. For a function f, when x is a member of the domain, the symbol $f(x)$ denotes the corresponding member of the range (e.g., $f(x) = x + 3$).
Fundamental Theorem of Algebra. The theorem that establishes that, using complex numbers, all polynomials can be factored into a product of linear terms. A generalization of the theorem asserts that any polynomial of degree *n* has exactly *n* zeros, counting multiplicity.

G

Geometric sequence (progression). An ordered list of numbers that has a common ratio between consecutive terms, e.g., 2, 6, 18, 54, . . .

H

Histogram. A graph that uses bars that have equal ranges of values.

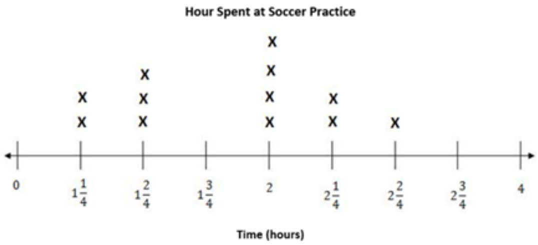
I

Identity property of 0. See Table 3.
Imaginary number. Any number of the form *bi*, where *b* is a nonzero real number and *i* is the square root of –1. See Illustration 1.
Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
Integer. All positive and negative whole numbers and zero.
Interquartile range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also: first quartile, third quartile.*
Inverse function. A function obtained by expressing the dependent variable of one function as the independent variable of another; that is the inverse of $y = f(x)$ is $x = f$ –1(y).
Irrational number. A number that cannot be expressed as a quotient of two integers, e.g., $\sqrt{2}$
It can be shown that a number is irrational if and only if it cannot be written as a repeating or terminating decimal.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

L

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. (Also known as a dot plot.)



Linear association. Two variables have a linear association if a scatter plot of the data can be well approximated by a line.
Linear equation. Any equation that can be written in the form $Ax + By + C = 0$ where A and B cannot both be 0. The graph of such an equation is a line.
Linear function. A function with an equation of the form $y = mx + b$, where m and b are constants.
Logarithm. The exponent that indicates the power to which a base number is raised to produce a given number. *For example, the logarithm of 100 to the base 10 is 2.*
Logarithmic function. Any function in which an independent variable appears in the form of a logarithm; they are the inverse functions of exponential functions.

M

Matrix (pl. matrices). A rectangular array of numbers or variables.
Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[3] *Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.*
Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.*
Measure of variability. A determination of how much the performance of a group deviates from the mean or median, most frequently used measure is standard deviation.
Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list; or the mean of the two central values, if the list contains an even number of values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.*
Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.
Mode. The most frequent value in a set of data.
Model. A mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real- world or mathematical objects, properties, actions, or relationships.
Monomial: An algebraic expression made up of one term.
Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0–100. Example: $72 \div 8 = 9$.
Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. *Example: 3⁄4 and 4⁄3 are multiplicative inverses of one another because 3⁄4 x 4⁄3 = 4⁄3 x 3⁄4 = 1.*

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[3] To be more precise, this defines the arithmetic mean.

N

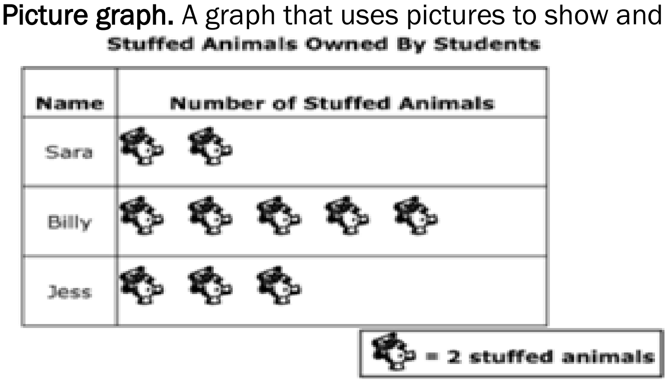
Network. a) A figure consisting of vertices and edges that shows how objects are connected; b) A collection of points (vertices), with certain connections (edges) between them.
Non-linear association. The relationship between two variables is nonlinear if the change in the second is not simply proportional to the change in the first, independent of the value of the first variable.
Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

O

Order of Operations. Convention adopted to perform mathematical operations in a consistent order. 1. Perform all operations inside parentheses, brackets, and/or above and below a fraction bar in the order specified in steps 3 and 4; 2. Find the value of any powers or roots; 3. Multiply and divide from left to right; 4. Add and subtract from left to right.

P

Partition. A process of dividing an object into parts.
Percent rate of change. A rate of change expressed as a percent. *Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.*
Periodic phenomena. Naturally recurring events, for example, ocean tides, machine cycles.
Picture graph. A graph that uses pictures to show and compare information.



Plane. A flat surface that extends infinitely in all directions.
Polar form. The polar coordinates of a complex number on the complex plane. The polar form of a complex number is written in any of the following forms: $r\cos\theta + r\,i\sin\theta$, $r(\cos\theta + i\sin\theta)$, or $r\,\mathrm{cis}\,\theta$. In any of these forms, r is called the modulus or absolute value. θ is called the argument.
Polynomial: The sum or difference of two or more monomials.
Polynomial function. Any function whose value is the solution of a polynomial.
Postulate. A statement accepted as true without proof.
Prime factorization. A number written as the product of all its prime factors.
Prime number. A whole number greater than 1 whose only factors are 1 and itself.
Probability distribution. The set of possible values of a random variable with a probability assigned to each.
Properties of equality. See Table 4.
Properties of inequality. See Table 5.
Properties of operations. See Table 3.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, testing for a medical condition).
Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also: uniform probability model.*
Proof. A proof of a mathematical statement is a detailed explanation of how that statement follows logically from statements already accepted as true.
Proportion. An equation that states that two ratios are equivalent.
Pythagorean Theorem. For any right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Q

Quadratic equation. An equation that includes only second-degree polynomials. *Some examples are* $y = 3x^2 - 5x^2 + 1$, $x^2 + 5xy + y^2 = 1$, and $1.6a^2 + 5.9a - 3.14 = 0$.
Quadratic expression. An expression that contains the square of the variable, but no higher power of it.
Quadratic function. A function that can be represented by an equation of the form $y = ax^2 + bx + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$. The graph of this function is a parabola.
Quadratic polynomial. A polynomial where the highest degree of any of its terms is 2.

R

Radical. The $\sqrt{}$ symbol, which is used to indicate square roots or *n*th roots.
Random sampling. A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects.
Random variable. An assignment of a numerical value to each outcome in a sample space.
Ratio. A relationship between quantities such that for every *a* units of one quantity there are *b* units of the other. A ratio is often denoted by *a*:*b* and read "*a* to *b*".
Rational expression. A quotient of two polynomials with a non-zero denominator.
Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers. See Illustration 1.
Real number. A number from the set of numbers consisting of all rational and all irrational numbers. See Illustration 1.
Rectangular array. An arrangement of elements into rows and columns.
Rectilinear figure. A polygon all angles of which are right angles.
Recursive pattern or sequence. A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.
Reflection. A type of transformation that flips points about a line, called the line of reflection. Taken together, the image and the pre-image have the line of reflection as a line of symmetry.
Relative frequency. The empirical counterpart of probability. If an event occurs N' times in N trials, its relative frequency is N'/N.
Relatively Prime. Two positive integers that share no common divisors greater than 1; that is, the only common positive factor of the two numbers is 1.
Remainder Theorem. If $f(x)$ is a polynomial in x then the remainder on dividing $f(x)$ by $x - a$ is $f(a)$.
Repeating decimal. A decimal in which, after a certain point, a particular digit or sequence of digits repeats itself indefinitely; the decimal form of a rational number. *See also: terminating decimal.*
Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Rotation. A type of transformation that turns a figure about a fixed point, called the center of rotation.

S

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.
Scatter plot. A graph in the coordinate plane representing a set of bivariate data. *For example, the heights and weights of a group of people could be displayed on a scatter plot.*
Scientific notation. A widely used system where nonzero numbers are written in the form $m \times 10^n$ where n is an integer, and m is a nonzero real number between 1 and 10, e.g., $562 = 5.62 \times 10^2$.
Significant figures (digits). A way of describing how precisely a number is written, particularly when the number is a measurement.
Similarity transformation. A rigid motion followed by a dilation.
Sine (of an acute angle). The trigonometric function that for an acute angle is the ratio between the leg opposite the angle when the angle is considered part of a right triangle and the hypotenuse.
Standard algorithm. A step-by-step approach to calculating, decided by societal convention, developed for efficiency. Flexible and fluent use of standard algorithms requires conceptual understanding.

T

Tangent. a) Meeting a curve or surface in a single point if a sufficiently small interval is considered. b) The trigonometric function that, for an acute angle, is the ratio between the leg opposite the angle and the leg adjacent to the angle when the angle is considered part of a right triangle.
Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.
Terminating decimal. A decimal is called terminating if its repeating digit is 0. A terminating decimal is the decimal form of a rational number. *See also: repeating decimal.*
Third quartile. For a data set with median M, the third quartile is the median of the data values greater than M. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range.*
Transformation. A prescription, or rule, that sets up a one-to-one correspondence between the points in a geometric object (the pre-image) and the points in another geometric object (the image). Reflections, rotations, translations, and dilations are particular examples of transformations.
Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.
Translation. A type of transformation that moves every point in a graph or geometric figure by the same distance in the same direction without a change in orientation or size.
Trapezoid. A quadrilateral with at least one pair of parallel sides. (Note: There are two definitions for the term *trapezoid*. This is the inclusive definition. For more information see commoncoretools.me/wpcontent/uploads/2014/12/ccss_progression_gk6_2014_12_27.pdf).
Trigonometric function. A function (as the sine, cosine, tangent, cotangent, secant, or cosecant) of an arc or angle most simply expressed in terms of the ratios of pairs of sides of a right-angled triangle.

U

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also: probability model.*
Unit fraction. A fraction with a numerator of 1, such as 1/3 or 1/5.

V

Variable. A quantity that can change or that may take on different values. Refers to the letter or symbol representing such a quantity in an expression, equation, inequality, or matrix.
Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.
Visual fraction model. A tape diagram, number line diagram, or area model.

W

Whole numbers. The numbers 0, 1, 2, 3, . . . See Illustration 1.

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Table 1. Common addition and subtraction situations[1]. This table shows the "situation types" or categories of word problems required by the standards in which the given number and the unknowns are in a variety of configurations. Darker shading [2] indicates the four ***Kindergarten*** problem subtypes. ***Grade 1 and 2*** students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2.

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
Put Together/ Take Apart[4]	**Total Unknown**	**Addend Unknown**	**Both Addends Unknown[3]**
	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
Compare[5]	**Difference Unknown**	**Bigger Unknown**	**Smaller Unknown**
	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?	Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?	Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Note: Additional guidance on addition and subtraction problem types can be found at Quick Reference Guide: Common Addition and Subtraction Situations (mass.edu)

[1]Adapted from Boxes 2–4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32–33).

[2]Adapted from Achievethecore.org :: Situation Types for Operations in Word Problems

[3]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[4]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[5]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2. Common multiplication and division situations[1] (Grades 3 – 5). This table shows the "situation types" or categories of word problems required by the standards in which the given number and the unknowns are in a variety of configurations

	Unknown Product **3 x 6 = ?**	**Group Size Unknown** ("How many in each group?" Division) **3 x ? = 18 and 18 ÷ 3 = ?**	**Number of Groups Unknown** ("How many groups?" Division) **? x 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are three bags with six plums in each bag. How many plums are there in all? Measurement example: You need three lengths of string, each six inches long. How much string will you need altogether?	If 18 plums are shared equally into three bags, then how many plums will be in each bag? Measurement example: You have 18 inches of string, which you will cut into three equal pieces. How long will each piece of string be?	If eighteen plums are to be packed six to a bag, then how many bags are needed? Measurement example: You have 18 inches of string, which you will cut into pieces that are six inches long. How many pieces of string will you have?
Arrays,[2] Area[3]	There are three rows of apples with six apples in each row. How many apples are there? Area example: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into three equal rows, how many apples will be in each row? Area example: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of six apples, how many rows will there be? Area example: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare[4]	A blue hat costs $6. A red hat costs three times as much as the blue hat. How much does the red hat cost? Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be three times as long?	A red hat costs $18 and that is three times as much as a blue hat costs. How much does a blue hat cost? Measurement example: A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

Note: Additional guidance on multiplication and division problem types can be found at Quick Reference Guide: Common Multiplication and Division Situations (mass.edu)

[1]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in three rows and six columns. How many apples are in there? Both forms are valuable.

[3]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[4]Multiplicative Compare problems appear first in Grade 4, with whole-number values, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Adapted from Achievethecore.org :: Situation Types for Operations in Word Problems

Table 3. The Properties of Operations Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.	
Associative Property of Addition	$a + b) + c = a + (b + c)$
Commutative Property of Addition	$a + b = b + a$
Additive Inverse Property of Zero	$a + 0 = 0 + a = a$
Existence of Additive Inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative Property of Multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative Property of Multiplication	$a \times b = b \times a$
Multiplicative Identity Property of One	$a \times 1 = 1 \times a = a$
Existence of Multiplicative Inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive Property of Multiplication over Addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The Properties of Equality Here a, b, and c stand for arbitrary numbers in the rational, real, or complex number systems.	
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \times c = b \times c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution Property of Equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The Properties of Inequality Here a, b, and c stand for arbitrary numbers in the rational or real number systems.
Exactly one of the following is true: $a < b$, $a = b$, $a > b$. If $a > b$ and $b > c$ then $a > c$. If $a > b$, then $b < a$. If $a > b$, then $-a < -b$. If $a > b$, then $a \pm c > b \pm c$. If $a > b$ and $c > 0$, then $a \times c > b \times c$. *If* $a > b$ and $c < 0$, then $a \times c < b \times c$. If $a > b$ and $c > 0$, then $a \div c > b \div c$. If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Table 6. Illustrative Example of an Alternative Algorithm vs. a Standard Algorithm

An **algorithm** is defined as a procedure for solving a mathematical problem in a finite number of steps that frequently involves repetition of an operation.

The standards require mastery of several U.S. Standard Algorithms for efficiency at specified grade levels. However, the standards are constructed so students' understanding and ability to use standard algorithms is predicated on the development of their conceptual understanding through experiences exploring and using a variety of alternative algorithms.

This table provides **one** example of an alternative algorithm for the operation of addition. High-quality curriculum materials (HQCM) will offer other alternatives when teaching addition. Furthermore, HQCM will include alternative algorithms for the operations of subtraction, multiplication, and division. The use of alternative algorithms aids in developing students' conceptual understanding of an operation.

One Example of an Alternative Algorithm for Addition*	Standard Algorithm for Addition (for efficiency)
356 +167 400 (Sum of hundreds) 110 (Sum of tens) 13 (Sum of ones) 523	11 (Regrouped 10 and 100) 356 +167 523

Note: Additional guidance on algorithms for addition and subtraction can be found at [Quick Reference Guide (QRG) - Algorithms Sept 2017 (mass.edu)]
Additional guidance for algorithms for multiplication and division can be found at [Quick Reference Guide: Standard Algorithms for Multiplication and Division (mass.edu)]

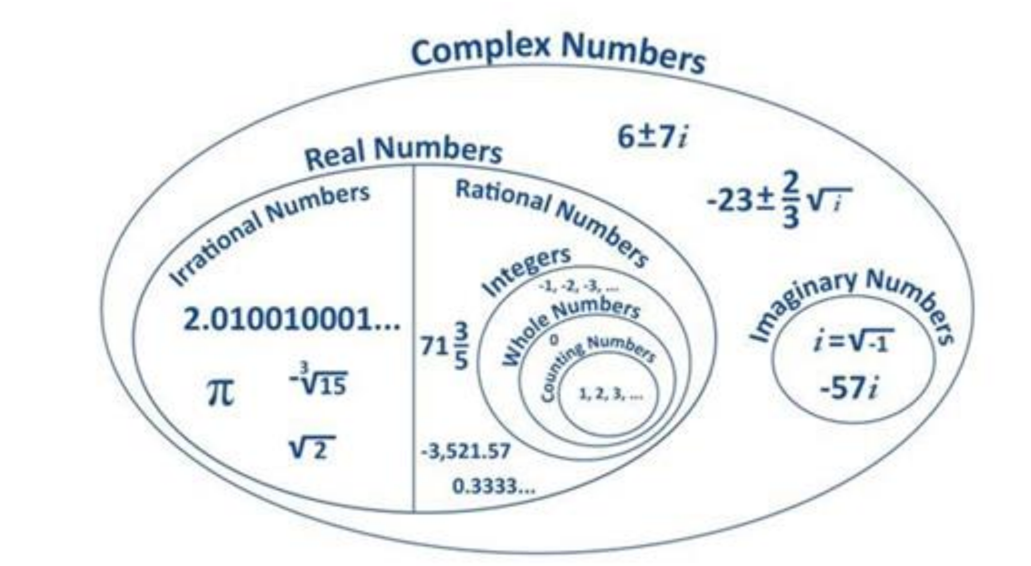
Table 7. Required Grade-Level Fluencies (Grades K – 6)

This table itemizes the fluencies required by the standards for grades K through 6. Fluency is **defined** as using efficient, flexible and accurate methods of computing. Table is adapted from Achievethecore.org :: Instructional Content Nav - Mathematics: Focus by Grade Level

Grade	Standard	Required Fluency
K	K.OA.A.5	Add/subtract within 5
1	1.OA.C.6	Add/subtract within 10
2	2.OA.B.2 2.NBT.B.5	Single-digit sums and differences (sums by memory by end of grade) Add/subtract within 100
3	3.OA.C.7 3.NBT.A.2	Single-digit products and quotients (products by memory by end of grade) Add/subtract within 1000
4	4.NBT.B.4	Add/subtract within 1,000,000
5	5.NBT.B.5	Multi-digit multiplication
6	6.NS.B.2 6.NS.B.3	Multi-digit division Multi-digit decimal operations

Illustration 1. The Number System*

The Number System is comprised of number sets beginning with the Counting Numbers and culminating in the more complete Complex Numbers. The name of each set is written on the boundary of the set, indicating that each increasing oval encompasses the sets contained within. Note that the Real Number Set is comprised of two parts: Rational Numbers and Irrational Numbers.



*Adopted from the Massachusetts Model Content Frameworks for Mathematics, 2017.