State of Rhode Island and Providence Plantations
DEPARTMENT OF EDUCATION
Shepard Building
255 Westminster Street

Deborah A. Gist
Commissioner

REVIEW OF MATH NECAP RELEASED ITEMS

September 14, 2012

Dear Math Educators,

Your colleagues at North Kingstown and Narragansett High Schools have generously provided this review and analysis of the October 2011 NECAP mathematics released items. Many of you have done similar things and this is meant to provide a different and, hopefully, confirming perspective for those efforts. This review will also provide a model for those of you who may not have reviewed the released items in this manner and as stated below may be a model for future efforts.

This review is intended to provide an additional piece of evidence about student math performance. Examining the released NECAP items, particularly from any one year, will not provide all of the answers about issues regarding student performance. This review is one additional piece of evidence that in many cases will confirm what you may already know about student performance from the classroom.

We greatly appreciate the effort extended by the teachers who volunteered to complete this review. They responded with a generous amount of their time this summer and we are glad to share this with you as you work with your students to support them in reaching math proficiency.

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NECAP Released Math Item Review

Dear Fellow Mathematics Educators,

At the request of the Rhode Island Department of Education (RIDE), and with their data regarding the Grade 11 2011 NECAP Mathematics student responses listed by achievement level, the review team endeavored to study the released items and student responses. We did this to try to see if we could find any common concept areas that were particularly problematic for students, especially those who struggle to achieve “Partially Proficient” status or above. The information we gathered, we believe, is largely the same conclusions that would have been reached by other mathematics educators given they had the time and data to do the job. Our hope in doing this project is that we saved you the time and effort in the analysis phase so that you can better use your time to devote to the next steps in working with your students in the short-term preparations for the 2012 NECAP mathematics test and the longer term goal of preparing for 2013. We were happy to initiate this professional review process and hope that in future years a similar undertaking can be accomplished on a wider scale through statewide participation via some type of web-based blog or Google document format.

Appendix A is a table format of each released item, a list of the concepts we noted could or should be needed for the solution, and the types of skill barriers we have noted both in our professional experience and in examining the student responses. As you look through this table, you will want to have a copy of the NECAP Released Items 2011 for Grade 11 Mathematics that can be accessed at the RIDE website.

John Boutcher, North Kingstown High School
Elizabeth LaPointe, Narragansett High School
Barbara Morse, North Kingstown High School

September 14, 2012

Appendix B is the document from RIDE which we used in our examination of the released items.

Appendix C is a more formal presentation of the information in Appendix A which was written to address the questions posed by RIDE in the Appendix B document.

Procedure

We examined the data provided by the Rhode Island Department of Education which showed the percentage of students in each NECAP scoring category correctly answering the released item question (Appendix A). Our strategy consisted of:

a) Determine the content knowledge necessary to answer the question
b) Note common student errors based a combination of our teaching experience and examination of the student responses provided on constructed responses
c) Consider potential instructional and content sequencing issues
d) Identify patterns of student skills and academic content in the most frequently missed questions
e) Suggest possible short and long-term strategies to address the most prevalent issues.

Results

The result of our examination of the released items is listed in a table format (Appendix B). This list was provided to a few outside readers in Google document form for review and comments; however, we hope that future reviews will be improved by having a state-wide set of reviewers working through a web-based communication forum.

The most frequent student skill and academic concept issues will not be surprising to most experienced math educators; however, we believe there is value in having our anecdotal information confirmed through the data. The most common issues found by our examination were:

- Proportional reasoning and relationships, particularly between measurements of one-, two- and three-dimensional objects.
- Reading comprehension of vocabulary, notation and ability to perform verbal to algebraic translations.
- Reading comprehension of graphs and tables.
- Operations of geometric figures on the coordinate plane particularly with nets and transformations.
- Following instructions, most notably “show your work or describe how you know”.

Possible short- and long-term strategies:

We all use practice problems from NECAP released items and other review packets, in the interest of shortening the time away diverted from our regular curriculum we will be using those that place emphasis on the most common problem areas listed above. We found some excellent extended problems to address these concepts on the Charles A. Dana Center website that could be purchased at a low cost (around $16 per content area) or for a short time can be downloaded for free after answering a short survey at http://www.utdanacenter.org/ccss/algebra-geometry.php. Their Algebra Chapter 2 unit on Function Fundamentals addresses many of the algebraic item weaknesses (non-linear tables, rates of change) and their Geometry Chapter 1 (Coordinate Geometry), Chapter 4 (Perimeter, Area, Volume) and Chapter 5 (Solids and Nets) could be very useful in parts even if time is not available for the whole units.

Longer-term solutions may mean reconsidering the scope and sequence of the content discussed in the Appendix A chart. As members of the Mathematics writing team working on the scope and sequence of the Core Curriculum State Standards in Rhode Island, we have noted that some of the issues will be addressed in the new curriculum. For example, a more extensive study of coordinate geometry and geometric transformations in Geometry and more emphasis on non-linear relationships and patterns in Algebra 1 is included in the CCSS.

Appendix A
Review Team's Analysis of Student Performance on Released Items
J. Boutcher, E. LaPointe, B. Morse – 8/2012

Item	Skills Needed	Common Student Errors	Potential Instructional Issues
1	Relative proportions Finding Cube roots Working "backwards" through formulas (i.e., students might know to take linear to volume but not volume to linear, or cube a number but not cube root)	Not knowing the formulas to convert linear measure to volume Dividing by three instead of cube root	Relative proportions are late in Geometry curriculum, thus may be rushed or not covered thoroughly. Little or no exposure to cube roots
2	Area of circle Segment addition/subtraction Distributive property for answer format	Forget to or improperly add segments. Difficulty seeing the areas shaded the same as unrelated. Simplifying the problem to 51pi thus not recognizing the answer	Students commonly do this problem with two areas, but not combinations of two or more.
3	3-dimensional nets 3-dimensional thinking Areas of polygons	Not recognizing three dimensions, thus only calculating two sides (Answer D) Miscounting the number of sides (Answers A and B)	
4	Ratios/proportions Squaring/square roots	This could be solved in a variety of ways; each with multiple steps Students accustomed to miles, feet, yards, etc. may attempt to convert the constants rather than solve for the variable	
5	Slope formula Parallel/perpendicular formulas Distance formulas	Visual students may try to graph to solve – access to graph paper? Sign errors in the slope if calculating	Working with alternative types of solutions to problems
6	Working with patterns in tables OR Working with sequences. Solving equations.	If trying to solve by repetition the problem is lengthy – students may give up. If calculating the pattern of adding 2 to each successive perimeter, students may not use correct y-intercept of 2. If solving by algebraically using slope:	If solving algebraically using slope: Students often work with two columns [x, f(x)], and three columns,[x , calculate f(x), f(x)], but rarely work with only two columns that contain x, calculate f(x), so they may not think to generate the third

		Because the directions say “table”, students may be confused about numerous “rules” in the perimeter column.	column to find 8x + 2. Once found, they would need to equate 8x + 2 = 6x + 50. Or, if students disregard the 6x, to find a pattern of 2x + 2, they would need to also disregard the 6x in the solution of 2x + 2 = 50, thus x = 24.
7	Working with intervals, interval notation	Vocabulary or reading proficiency (“less than zero” means slope is negative v. looking below zero on x axis) Vocabulary “function” and “rate of change”	Intervals is usually addressed in Algebra 2 – may be after test.
8	Solving systems of equations. Converting phrases into expressions.	Reading proficiency – seeing the 5180 in the directions as part of the information needed for system. Students often reverse “as many” problems, for example, three times as many x as y may be written 3x=y instead of x=3y	Alternate directions – don’t follow textbook formats that put all information in bullets. Experience converting tricky phrases like “as many” into algebraic
9	Converting standard form to slope-intercept form OR finding x/y intercepts from standard form. Reading labels on graphs. Reading graphs.	Lack of experience or calculation errors in converting. Error in identifying which axis represents x values and y values according to labels.	Relate slope to correlation (for example, the more raising you buy the less peanuts you can get is a negative correlation)
10	Correlation	Middle school content (we saw this as related to Question 9)	Few students missed this question, but those that missed it may have been too long since the material was covered.
11	Visual relationships (“Other” has biggest piece of circle, “Other” is twice the size of “Medicine”, “Law” is 1/5 of “Other” and ¼ of “Education”, etc) Visual reasoning	Middle School content	

12	Same reasoning skills as #11, but students might not recognize this. Calculating denominator as sum of trials v. 100 for percent.	Overthinking between experimental and theoretical probability. Students choose “Percent” denominator of 100 (Answer A) Students incorrectly calculator denominator as part Red/(Blue + Green) instead of R/(B+G+R) or students incorrectly reduce OR Common error of using cancellation to reduce R/(R+G+B) to R/(G+B).	Middle school content may not be reviewed in high school.
13	Reflections Vocabulary: “Image” and “reflection”	Reflecting over y axis instead of axis of symmetry at x = -2. Sign errors in writing coordinates.	
14	Working with nets. Same reasoning skills as #3, but in reverse format (Net to 3-dimensions)	Not seeing the base as section of trapezoid. Confusion between surface area and volume. Forgetting unit notation	More presentation of these types of problems is needed, working in both directions (linear measure to volume, volume to linear, volume to surface area, etc)
15	Sequences and patterns OR Proportions	Students will see fractions and not attempt the problem. Students try to work as a table and calculate slope with fractions (erroneously seeing table as linear relationship) Students difficulty working with fractional proportions 5/(1/10)=10/(1/20)	Problem is extremely difficult if student does not recognize the pattern of x and 1/(2x) that results in x / [1/(2x)]
16	Substitution Order of operations	Improper substitution Improper cancellation of values in numerator/denominator (see #12)	Strategies: color coding or labeling of values for substitution, writing out values of variables, reinforcing putting all expressions into parentheses before calculating.

17	Translating phrases into expressions or equations. Reading proficiency.	Reversing values (L=3-2w versus L=2w-3). Related to #8 Writing in terms of area vs perimeter.	
18	Writing fractions Probability complements	Not writing probability as %, ratio, or decimal (e.g. just wrote 9980) Not using complement of 9980/10,000	
19	Numeracy (estimation) Square root approximation Radical operations Reading directions	Students may not recognize square root of ½ is square root of 1 divided by square root of 2. Once 3/rad 2 is found, students may divide 3 by 2 rather than 3/1.4	
20	Coordinate geometry: Using distance formula to find length of polygon sides OR Right triangles: using the 45-45-90 property using sides of polygon as hypotenuse. Reading directions: Show your work	Miscalculation of distance formula. If using right triangle, not recognizing special triangle property thus creating calculation complications with radicals. Not showing work or writing explanation	
21	Adding like terms. Substitution. Order of operations.	Students incorrectly calculated 12(5)^2 as 60^2.	
22	Translating phrases to expressions. Finding percent of fraction. Reading proficiency.	In part c, students may have taken 40% of 60 from part b instead of reading that 6 people represent 40% of the total receiving gifts (thus 15 received gifts).	This question was very poorly worded – using “services as excellent” in part c might have led students to connect answer b to answer c – a not unexpected result as problems presented often require one response to answer the next part. For part B perhaps diagraming like in #3 would have helped students see that 80 is less than ½ of 200, thus answer to part c could not be more than 80 as at least one student answered.

23	Vocabulary: exponentially Reading/interpreting graphs Incremental measurements Percent of increase/decrease Estimation or logarithms for part c	Part a: Not using y intercept to find increase in first 10 year period. Part b: The wording indicates that students should calculate each ten year interval separately; exponentially increasing means different rate of change over each 10 yr interval. (Students may be perplexed that the last 4 year interval calculates as 43% increase despite the increased slope in the graph compared to previous intervals of 50%, 50% and 55%). If you use exponential regression f(x) approximates 80(1.5)^x. By graphing the values in a table and using STAT plot, one can accurate see that the expected answer is 50% increase per 10 year interval.	These types of problems, using exponential regression is usually presented in Algebra 2 after the NECAP test. Perhaps in Algebra 1 we need to cover other types of regression when we do linear. 73% of the proficient with distinction answered correctly, probably because they typically complete Algebra 2 curriculum before the NECAP.

Appendix B

RIDE Document – Performance on Grade 11 NECAP Mathematics Released Items by Achievement Level

C. DePascale, NCIEA, 7/17/2012

NECAP Grade 11 Mathematics - Fall 2011 Released Items

Determining Knowledge and Skills that distinguish between students performing at different achievement levels

Performance on Grade 11 2011 NECAP Mathematics Released Items by Achievement Level						
Released Item Number	Item Type	Percent Correct or Percent of Points*	Substantially Below Proficient	Partially Proficient	Proficient	Proficient with Distinction
1	MC	25%	14%	16%	42%	92%
2	MC	35	11	28	70	95
3	MC	44	22	41	72	93
4	MC	31	22	23	45	90
5	MC	26	20	18	36	84
6	MC	60	36	65	85	98
7	MC	37	18	28	65	96
8	MC	41	18	42	67	87
9	MC	28	18	22	44	86
10	MC	72	50	81	92	99
11	MC	79	64	87	93	97
12	MC	53	26	58	84	96
13	SA1	37	9	38	71	93
14	SA1	14	1	7	32	78
15	SA1	19	3	12	41	86
16	SA1	56	23	66	89	98
17	SA1	24	1	15	59	90
18	SA1	37	10	39	70	93
19	SA2	13	6	9	21	74
20	SA2	13	<1	5	33	88
21	SA2	37	9	38	69	91
22	CR4	36	11	35	66	95

23	CR4	26	12	26	41	73

*For multiple-choice items 1-12 and short answer 1-point items 13-17, figures indicate the percentage of students responding correctly. For short answer 2-point and constructed response 4-point items 19-23, figures indicate the percentage of possible points earned by students in each achievement level category.

Reading the Table

The table presents information on student performance on the 23 released items from the grade 11 2011 NECAP Matheamatics test. The first column provides the Released Item Number corresponding to the item numbers in the released item materials and item analysis reports. The second column identifies the item type, either multiple-choice (MC), 1-point short answer (SA1), 2-point short answer (SA2), or 4-point constructed response (CR4). The third column provides the overall percent correct or percent of possible points student earned on each item. For 1-point MC and SA1 items, the figure reflects the percentage of students who answered the item correctly. For the 2-point SA2 and 4-point CR4 items, the figure indicates the percentage of possible points earned. For example, a score of 1 on an SA2 item would equal 50% of the possible points on the item. Similarly, a score of 1 on a 4-point CR4 item would equal 25% of the possible points on that item.

The figures in columns 4 through 7 of the table show the performance on each of the grade 11 NECAP mathematics released items based on students' overall achievement level. For example, the table indicates that 14% of students whose performance on the Mathematics test was classified as *Substantially Below Proficient* answered Released Item 1 correctly. That figure can be compared to 16% of students whose performance was classified *as Partially Proficient*, 42% for students with *Proficient* performance and 92% for students performing at the *Proficient with Distinction* level.

Interpreting the Table

At a quick glance, several pieces of information are available from the table

- From column 3, it is apparent that there is wide variation in student performance across the items – ranging from just over 10% on some of the SA1 questions to greater than 70% on two of the MC items
- In general, students in higher achievement levels perform better on the item.
- The vast majority of students at the *Proficient with Distinction* level responded correctly to almost all of the items, but there were only two items on which even a majority of students performing at the *Substantially Below Proficient* level responded correctly (items 10 and 11).

Looking more closely, we can begin to identify items on which students at one level are successful, but that the majority of students at the preceding level cannot answer correctly. For example, consider Item 1:

- 92% of students at the *Proficient with Distinction* level answered this item correctly.
- Only 42% of students at the *Proficient* level answered this item correctly.
- We can argue that the knowledge and skills required to answer this question distinguish students at *the Proficient with Distinction* level from students at the *Proficient* level
- A review of the content of the item can help to determine whether that knowledge and skills includes specific mathematics content knowledge, problem solving skills, persistence, etc.
- Looking across several items, we hope to identify sets of skills that distinguish performance between the levels.

As a general guideline, we use to following criteria to identify items that distinguish between performance at adjacent achievement levels.

- The item is answered correctly by at least 65% of the students at the higher level.
- The item is answered correctly by fewer than 50% of the students at the lower level.
- There is at least a 20 percentage point gap in the percentage of students answering the item correctly at the two levels.

Applying those criteria to Released Item 1, we find:

- 92% of students at the Proficient with Distinction level responded correctly (more than 65%)
- Only 42% of students at the Proficient level responded correctly (fewer than 50%)
- The gap between 92% and 42% is 50 percentage points (greater than 20).
- Therefore, we conclude that this item distinguishes between students performing at the *Proficient and Proficient with Distinction* levels.

Note that on Item 1 the gap between performance at the *Partially Proficient* level (16%) and the *Proficient* level (42%) is also greater than 20 points (i.e., 42 – 16 = 26 points) and clearly fewer than half of the *Partially Proficient* students responded correctly (16%). However, because fewer than half of the students at the *Proficient* level responded correctly, although it is clear that Proficient students are more likely to answer the item correctly, we do not claim that this item is one that distinguishes between performance at those two levels

Not all items on a test will meet these criteria and be useful in identifying skills that distinguish between achievement levels. Some easier items (e.g., Items 10, 11) will be answered correctly by a majority of students performing at the lowest level on the test. Although less common, there may also be items on which fewer than 65% of students at the highest achievement level responded correctly. There will also be items such as Item 12 on which there is clearly increasing performance across the achievement levels, but the criteria are not quite met.

In terms of providing information that might be helpful to inform instruction, these criteria should be regarded as guidelines and not hard and fast rules. If an item such as Item 12 comes very close to meeting all three criteria, it can still provide some useful information.

Applying the criteria to all of the items, we can identify those items that distinguish between performance at adjacent achievement levels.

<u>Substantially Below Proficient v. Partially Proficient</u>

Item 6

Item 16
Item 12*

Partially Proficient v. Proficient
Item 2
Item 3
Item 7
Item 8
Item 13
Item 17
Item 18
Item21
Item 22

Proficient v. Proficient with Distinction
Item 1
Item 4
Item 5
Item 9
Item 14
Item 15
Item 19
Item 20
Item 23

Including Item 12 in the list of items that distinguish between performance at the *Substantially Below Proficient* and *Partially Proficient* levels, we account for 21 of the 23 released items – that is all items except Item 10 and 11 which were answered correctly by the majority of students at the *Substantially Below Proficient* level. As mentioned previously, this is not always the case.

The next step in the process is for people with content and instructional expertise to review the items in an attempt to identify the knowledge and skills needed to respond to the item. The process requires professional judgment and might not be straightforward. There are always some

items on which it is impossible to determine precisely what made the item difficult for students at a particular level. Working across the entire set of released items from this year and previous years, patterns of knowledge and skills should emerge.

One caution in performing these analyses and attempting to draw conclusions and make interpretations is that there has to be some understanding of the motivation and effort displayed by the students responding to the test items.

In conclusion, let's work through one item as an example of the process. Consider Item 1 once again, which appears to distinguish between performance at the *Proficient* and *Proficient with Distinction* levels. Recall that fewer than 20% of students at the lower two achievement levels answered this item correctly.

1 The ratio of the volumes of two cubes is 1:64. What is the ratio of the edge lengths of the two cubes?

A. 1:4
B. 1:8
C. 1:16
D. 1:64

There is no complex mathematics knowledge required to answer this item correctly. The key information that is necessary in terms of mathematical knowledge is to know the formula for the volume of a cube – and that is provided on the Mathematics Reference Sheet in the general form as a Rectangular Prism. With that information, the problem can be solved fairly quickly with the simple technique of setting up a table and examining the ratios of volumes and sides.

Cube	Length of Edge	Volume
A	1	1
B	2	8

C	3	27
D	4	64

After completing just four rows in the table, it can be seen that when the ratio of volumes is 1:64 (Cube A: Cube D), the ratio of their edge lengths is 1:4 (Option A).

Interpreting performance on this item, therefore, it is necessary to attempt to determine how much student performance was influenced by the mathematical knowledge required (i.e., volume of a cube, understanding of ratio, meaning of 'edge') and how much it was influenced by the context of the item and any problem solving skills that might be required. You may ask and discuss questions such as:

- Would a student at the *Substantially Below Proficient* level be able to answer the question: *What is the volume of a cube with an edge length of 4?*
- Would a student at the *Partially Proficient* level be able to answer the question: *What are the edge lengths of two cubes that have volumes of 1 and 64?*
- Would a student at the *Proficient* level be able to answer the question: *What is the ratio of the volumes of two cubes with edge lengths of 1 and 4?*

Discussion of questions such as these by people with content and instructional expertise can begin to inform instructional strategies and approaches to move students toward proficiency.

Appendix A: Fall 2010 Released Items

Performance on Grade 11 2010 NECAP Mathematics Released Items by Achievement Level						
Released Item Number	Item Type	Percent Correct or Percent of Points*	Substantially Below Proficient	Partially Proficient	Proficient	Proficient with Distinction
1	MC	57%	30%	60%	84%	98%
2	MC	33	21	29	46	82
3	MC	58	42	56	77	94
4	MC	43	30	39	59	91
5	MC	63	42	68	83	97
6	MC	43	16	39	75	96
7	MC	37	20	29	60	93
8	MC	51	37	47	70	96
9	MC	74	46	84	97	99
10	MC	86	74	91	96	98
11	MC	61	37	63	85	97
12	MC	35	22	31	52	90
13	SA1	39	14	41	63	90
14	SA1	14	1	6	33	80
15	SA1	29	9	21	53	93
16	SA1	11	<1	4	25	74
17	SA1	20	2	10	44	90
18	SA1	37	7	35	71	95
19	SA2	27	3	18	59	95
20	SA2	20	6	16	37	75
21	SA2	17	1	6	41	90
22	CR4	15	3	10	31	77
23	CR4	37	10	33	68	95

*For multiple-choice items 1-12 and short answer 1-point items 13-17, figures indicate the percentage of students in each achievement level category responding correctly. For short answer 2-point and constructed response 4-point items 19-23, figures indicate the percentage of possible points earned by students in each achievement level category.

Appendix C

Summary of Responses to Appendix A Performance Level Discrepancies

Substantially Below Proficient v. Partially Proficient

Item 6:

The primary discrepancy between the level 1 and the other levels is the ability to express patterns in algebraic form, utilize strategies to find information not expressly given and academic persistence in solving problems.

Strategy A: Using repetition (determining how many times to add 2 to $6x + 4$ to yield $6x + 50$) would be confusing to students who calculated 23 inches as the answer because they did not start at the width = 0 inches measure of $6x + 2$ for perimeter. Students with experience working with two-column tables may see the perimeter as the function rule and incorrectly calculate that they should add 2 to each perimeter, but may not see that because of the $6x\ term$ every increase of 1 unit in x results in an additional increase of 6 in the perimeter, yielding an overall increase of 8 in the perimeter for every 1 unit increase in the width. This would be a common error for students

Strategy B: Using the pattern in the tables without regard to the 6x term, yields (1, 4), (2, 6), etc. yielding a slope-intercept expression of 2x + 2. Similarly disregarding the 6x term in the expression $6x + 50$, then $2x + 2 = 50; x = 24$.

Strategy C: Students who have developed strategies such as extending tables to find relationships and work with linear and non-linear functions may extend the table by evaluating the width, yielding (1, 10), (2, 18), etc. to find the expression of $8x + 2$. Or, because the dependent value of perimeter is an expression rather than a constant, the student may calculate $(6x + 6) - (6x + 4)$ for $\Delta y/\Delta x$ to find a rate of change of 2 and that the y-intercept would be $6x + 2$ yielding a slope-intercept equation of $y = 2x + 6x + 2 = 8x + 2$. Solving would then be the intersection of $8x + 2 = 6x + 50; x = 24$.

Item 16:

The primary discrepancy between levels is the common issue of inability to accurately follow order of operations in both hand calculations and in entering information into calculators. Failing to input parentheses around the expression $(s^2 + a^2)$ when calculating on a calculator will yield 2250001 instead of 64.

Another frequent error is incorrect reduction or "cancelling" of values in the numerator and denominator, as in $\frac{s^2}{s^2+a^2} = 1/a^2$.

Item 12:

Writing and working with proportional relationships was one of the most problematic content issues found across the test. This content is most typically middle school material, thus Level 1 students may be working below or at the middle school level may not be experienced in finding probability in data tables.

Other sources of errors include:
Using 100 as the denominator for all probability ratios (answer A)
Using theoretical probability in place of experimental (answer B)
Incorrect use of denominator or incorrectly reducing, i.e. "cancellation" (answer C)

Partially Proficient v. Proficient

Item 2:

We saw this as an issue of depth of knowledge in the problems most often presented to students as level 2 and level 3, and a difference in the ability to analyze given illustrations. For example, students at Level 2 have experience with difference of area problems ("donut hole") where one area of a polygon or circle is subtracted from a larger polygon or circle area. These are most often represented as shaded area less un-shaded areas, thus the similar shading of the exterior ring and innermost circle could have lead to some confusion. More likely, Level 3 students would have more experience in working with more than two polygons, as in the difference of <u>sums</u> of polygon or circle areas being subtracted from a larger area.

Item 3:

This problem requires students to utilize nets or 3-dimensional thinking to include the surface areas not visible in the 2-dimensional representation. The ability or propensity to accurately read instructions is another potential discrepancy between level 2 and 3 students. Combining these two scenarios, level 2 students might have been more likely to try to include the surface areas of the sides of the building (dimensions unknown) for an answer of 8 x 1200 (answer A), or include only the two sides of the roof visible in the diagram for 2 x 1200 (answer D).

Item 7:

Interval notation and piecewise functions may not be introduced until Algebra 2, thus students not having completed Algebra 2 may not be very proficient in referencing pieces of graphs or the rate of change of those pieces in terms of the x-axis. Also, students at Levels 1 and 2 sometimes have difficulty interpreting phrases such as "*rate of change* that is *less than zero*", thus instructions given at those levels might more frequently be phrased as "*negative slope*".

Item 8:

Textbooks written for lower level students may be more apt to put all of the applicable information to solve a problem in the bullets of the problem, thus Level 2 students might not have seen $5180 in the instructions as part of the equation formulated by the first two bullets ($5x + 8y = 5180$). Secondly, in an error commonly seen in novice students when translating phrases in algebraic expressions, Level 2 students would be much more likely to translate "four times as many tickets for balcony seats", *x*, "as for orchestra seats", *y,* as $4x = y$ rather than the correct $x = 4y$. Finally, students who are not yet proficient in Algebra 1 concepts are less likely to solve systems of equations by substitution, as is required to find the solution of 740 balcony seats and 185 orchestra seats. Those correctly using substitution, but with the previously mentioned incorrect equation of $4x = y$, would find the answer of 140 balcony seats (answer A).

Item 13:

Geometric transformations on a coordinate grid was one of the most common weaknesses of all students, so a problem involving a reflection on an axis of symmetry was not surprising between levels 2 and 3. The error noted on the open responses were related to reflecting the figure over the y-axis instead of the axis of symmetry (x = -2), and sign errors related to writing the coordinates.

Item 17:

The error found in this problem is strongly related to the error in item 8 of incorrectly translating phrases into algebraic expressions. As seen in one sample response, students working at levels 1 and 2 are more likely to incorrectly translate the phrase “3 feet less than twice the width” as $3 - 2w$ rather than correctly as $2w - 3$. Students at level 2 may also be more prone to confuse area and perimeter formulas (using 2 widths multiplied by 2 lengths) and improper use of notation as in $2w - 3w$ instead of $(2w - 3)w$.

Item 18:

This was a more sophisticated version of item 12, using probability in sentence form instead of table form and requiring the use of the complement. As with other problems, the sample responses showed that students struggle with notation (neglecting to write the denominator in a ratio) and computation when reducing fractions.

Item 21:

In part a of this question, students displayed difficulty in combining like terms, such as adding exponents of variables, as in $3x^2 + 4x^2 + 5x^2 = 12x^6$ or by adding $2xy + 5xy = 8x$ This is more frequently seen in novice Algebra 1 students.

In part b of this question, sample responses for item 21 showed errors made involved use of order of operations, especially when substituting and evaluating. Students incorrectly calculated $12(5)^2 = 60^2$. This is related to the errors noted in item 16.

Item 22:

Part a of this problem was a very low level question requiring students to reduce the fraction 80/200 to 40/100 and write the response as 40%. Only a third of the sample responses showed errors when students used the reciprocal 200/80. Part b of this problem required students to find 75% (or ¾) of the answer in part a, which 5/6 answered correctly as 60, although one student wrote 60% instead of 60 people.

We suspect that the high rate of correct responses for part a and b put this item in the list for “Partially proficient v. Proficient” list instead of the “Proficient v. Highly Proficient” list. Part c of the problem has very complex wording in which the key information, 40% representing 6 users, was missed in several of the samples. However, one of the primary reasons for the discrepancy between level 3 and 4 was more likely due to the missing explanations when students disregarded the instructions “Show your work or explain how

you know”. In the experience of the reviewers, students below level 3 often need to be reminded that showing work and detailing explanations is much as part of the solution as the numerical answer.

Proficient v. Proficient with Distinction

Item 1:

These types of proportional conversions are typically presented in the latter part of Geometry courses, after the foundations of polygon properties, surface area and volume are completed. Due to the pacing difference between regular and accelerated courses, we suggest that the difference between Level 4 students and below Level 4 students is the amount of time and practice devoted to the ratio conversions between linear units, square units and cubic units. For example, if the length to length ratio between two rectangular bases is $a:b$, the area ratio would be $a^2:b^2$ and the volume ratio would be $a^3:b^3$. Working from a given volume ratio to the ratio of the lengths would require students to use cubic roots in solving equations, which is introduced in Algebra 1 but sometimes not reached as a proficiency until Algebra 2. Students in accelerated courses would be most comfortable using the cube root of 1:64 to yield 1:4.

Without this type of informational background, students might need to create a table of values in order to find the proportional relationships between the side lengths and volumes. We don’t think this is a strategy that would be obvious to Level 1, Level 2 students. An instructional issue that may need to be addressed is the compartmentalization of strategies wherein students think of tables of values as algebraic and therefore might not think to apply one to a geometric problem. Level 3 students may be more inclined to utilize a table strategy, but due to experience with ordered pairs are likely to only see the horizontal relationships 1: 1, 2:8, etc., and might find the correct answer in reducing 4:16 to 1:4.

Item 4:

Item 1 and Item 4 are somewhat related in that they require students to form ratios or proportions using objects with different dimensions. Students at all levels practice linear conversions in unit categories (inches to yards, feet to miles, centimeters to meters, etc.), and less frequently linear measures across unit categories (feet to meters, inches to centimeters). As noted in Item 1 comments, students may be less practiced in ratios between linear and non-linear measurements. Also, in this problem students might not recognize the unfamiliar “rods” as a linear measure and “acres” as two-dimensional or squared measurement. In order to solve this problem students must connect the following steps:

Step 1: Given, 1 mi = 320 rods

Step 2: Squaring both sides, 1^2 sq miles = 320^2 sq rods
Step 3: Given: 1 sq mi = 640 acres
Step 4: By substitution, 320^2 sq. rods = 640 acres
Step 5: Dividing both sides, 160 sq. rods = 1 acre.

Item 5:

Unless a student graphs the points and measures the angles, knowledge of parallel and perpendicular slope relationships is required to solve this problem. Student errors could include arithmetic errors in calculating the slopes and failure of students to recall that if a line has slope $m = a/b$ then the perpendicular slope is $m = -b/a$. The discrepancy between Level 4 and the other levels was surprising to us.

Item 9:

Related to Item 8, students must first create an equation using the given information, in this case $x + 2y = 6$ where x = cost per pound of raisins and y = cost per pound of peanuts. Unless students are familiar with using standard form to find x- and y-intercepts, they must convert the equation to slope-intercept form $y = -1/2x + 3$. Working with literal equations and these types of equation conversions can be difficult for students, and students frequently make sign errors (answers A and C) or forget to divide all terms by the coefficient of y (potential for answer D). As in Item 8, students may have reversed the relationship of the variables or misread the diagram, this time interpreting $x + 2y = 6$ for the graph so that y = cost per pound of raisins and x = cost per pound of peanuts.

Item 14:

This item required students to convert a two-dimensional net to calculate a 3-dimensional measurement of volume. As we saw in items 1 and 3, translating between two- and three-dimensional objects and measurements poses problems for many students. Students in Level 1 and 2 might have been able to decipher the terminology of “triangular prism” to mean that the base are was triangular and in the Pythagorean dimensions of 3,4,5 that form the lengths of the edges seen in the rectangular surfaces of the prism. Also as demonstrated in other items, students have difficulty in applying the correct formulas. In one student response the calculations were correct but for were for surface area.

Item 15:

The sample responses were all either accurate, or close to accurate with errors in notation. For example, $y = 1 \div 2x$ instead as opposed to $y = 1 \div (2x)$. Another student sample wrote the answer in the form of an expression instead of an equation, thereby losing points. It was our suspicion that many students either did not attempt to answer at all either because the relationship involved the use of fractions or because the relationship was not linear and thus difficult to solve unless the pattern was recognized instead of calculated.

Item 19:

This item required the skills we have previously noted as problematic in other items such as proportional relationships (items 1, 4, 15, 22), working with radical values (item 1), and the reluctance to show or explain work (items 20, 22, 23). In addition, students needed to be able to estimate the value of $\sqrt{1/2}$ or $1/\sqrt{2}$ as approximately .7. Any one or a combination of these issues excluded the majority of the students below Level 4 from answering this question correctly.

Item 20:

Errors by students in level 1 or 2 were most likely due to lack of experience with the concepts needed to solve the problem (distance formula, Pythagorean theorem or special right triangle relationships), or experienced difficulty in the calculations in using the distance formula or perimeter formula. This item demonstrated the difference between level 3 students who are more likely to use explicit strategies (distance formula) versus level 4 students who can devise implicit strategies (extending the lengths to form right triangles). Some students used the distance formula to find the lengths of the diagonal sides and then added those lengths. Because these calculations can be lengthy and prone to sign errors, several errors were found in the sample responses of students using this strategy. Students extending the polygon to form the sides legs of right triangles used the Pythagorean theorem or special right triangle properties to quickly find side lengths $7\sqrt{2}$ and $\sqrt{2}$, thus the perimeter was $16\sqrt{2}$, or in unsimplified form $2\sqrt{2} + 2\sqrt{98}$.

Item 23:

Part a – As we would have expected, 5/6 sample responses correctly gave $40 as the increase between the initial value of $80 and the ending value of $120 for the first 10 year increment. The error noted measured from the graph origin of $0 to $120. The response to this question was accessible to almost all students.

Part b – Because this function is increasing exponentially, the measurement of each interval posed problems for most students. Percentage of increase is covered in middle school curriculum [% increase = (New value – original value) ÷ original value], and rate

of change is covered in Algebra 1 (change in dependent value ÷ change in independent value). Either of these strategies could have been used to find the increase in each time increment; however, students below level 4 are probably not as adept at using these strategies in non-linear relationships such as the one depicted in the graph. We did note that some students using the % increase method may be confused at the uneven calculations between the 3rd and 4th ten-year periods as was expressed in one of the sample responses.

Part c – The solution for this part depended mostly on the student's calculations in part b. Almost all of the samples showed that students applied their rates of increase to calculate the 5th, 6th and 7th year interval, and those students who had found 50% increases in the intervals in part b were able to provide the correct answer.