GEOMETRY AND MEASUREMENT GRADE BY GRADE

Kindergarten

At kindergarten level, the Grade Level Expectations specify four stems that address (1) two-dimensional geometric figures, (2) measurement, (3) time, and (4) spatial relationships.

Two-Dimensional Geometric Figures

The first stem relates to two-dimensional geometric figures. Two-dimensional objects such as the face of a chalkboard, or the surface of a wall or the face of a pattern block do not exist in isolation and need to be abstracted from some actual three-dimensional object. The same goes for one-dimensional objects such as the rim of a cup or the edge of a box. This is because the world that we live in is made up of three-dimensional objects rather than two-dimensional objects. Thus, the two dimensional shapes that kindergarten students will meet will be found in three-dimensional objects such as balls, cubes, and cylinders.

In kindergarten, students need to be concerned with the shape of objects and learning the different characteristics of different shapes. Asking students to sort or classify shapes provides a mechanism for focusing on the shape of an object. There is no single right way to classify shapes. It is important that students classify shapes according to the characteristics that stand out for them. Classification skills develop gradually and very young children learn to recognize a great many shapes without being formally taught. Therefore, they may classify shapes according to their color or function rather than their shape. For example, young students might classify shapes as those which hold things and those which do not. In kindergarten any kind of classification system is fine. It is important that students are having plenty of opportunities to experience hand-on three- and two-dimensional objects. Here is an example:

Shape Sorter

Find the shapes in the picture and color them:

Measurement

The second stem relates to measurement. Young children begin to grapple with measurement issues long before they enter kindergarten.

When teaching how to measure length the following four stages are useful and widely used by teachers: However, it is unlikely that students will conquer each of these stages before the end of kindergarten. It seems reasonable, however, for kindergarten students to have accomplished the first stage before graduating to first grade.

Stage One: Direct Comparison

For example, comparing the length of two pencils. Students are given a pair of pencils and asked to decide which pencil is longer or shorter. The students measure the pencils by direct comparison; that is, they align the ends of each pencil and decide which pencil is longer or shorter.

Stage Two: Indirect Comparison

For example, suppose that you want to compare the length of two objects that cannot be compared directly, you can use the length of some other thing that can be compared to both objects directly.

For example, suppose you want to compare the adjoining two sides of a rectangular table. Put an unmarked and disposable tape to one of the sides that you want to compare and cut the exact length of the side. Then place the cut length of tape to the other side of the table for comparison. This will allow you to decide which is longer.

Stage Three: Measurement By Using Non-Standard Units

For example to represent the length of a side of a table, take a post card or index card, and count the number of cards needed to represent the length of the table. The non-standard unit is card length.

Stage Four: Measurement By Using Standard Units

Measure the length of a side of a table using a ruler calibrated in inches or centimeters.

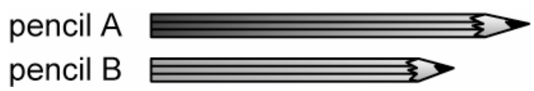
These four stages are designed to enable students to understand the principles of measuring length.

It is probably fair to expect kindergarten students to have accomplished Stage One. That is, in kindergarten, young children should have plenty of opportunity to compare quantities, especially lengths, by direct comparison. However, also in kindergarten young children should have plenty of opportunity to make indirect comparisons of length and capacity (Stage Two) and indirect measurements of the length or capacity of an object using non-standard units (Stage Three).

Even at the kindergarten level, it is important that students understand that a measurement must be accompanied by a unit. Students must learn that a number without a unit is not a measurement. Of course it is important for the teacher to realize that the unit does not need to be a standard unit, a nonstandard unit is fine.

Here are examples of some activities that can be used in kindergarten.

Which Pencil Is Longer?



Time

The third stem relates to time. During kindergarten students must be given the opportunity to determine elapsed and accrued time as it relates to calendar patterns (i.e., days of the week, yesterday, today, and tomorrow). Students should be able to sequence events in a day and identify a clock and calendar as tools that measure time. Here is an example:

Time

1		Afternoon	Evening
2	Yesterday	Today	
3		Week	Year

Which word goes in each space?

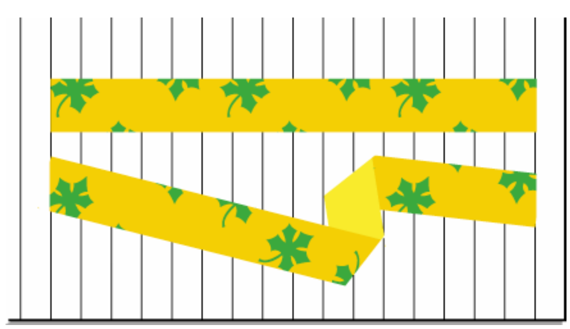
Tomorrow Day Morning

Spatial Relationships

The fourth stem relates to spatial relationships. During kindergarten, students must be given the opportunity to demonstrate an understanding of spatial relationships by using positional words such as *above*, *behind*, or *in front of* to locate and describe where an object is found in the environment. Here is an example that involves many parts of the measurement stem:

Ribbons

Which ribbon is longer, the one on top or the one on the bottom?



Grade 1

At the grade 1 level, the Grade Level Expectations specify seven stems that address (1) two-dimensional geometric figures, (2) three-dimensional geometric figures, (3) congruence, (4) geometric measurement, (5) measurement, (6) time, and (7) spatial relationships.

Two-Dimensional Geometric Figures

The first stem relates to two-dimensional geometric figures. For example, it is important that students in grade 1 become able to distinguish triangles, squares, and circles. Students learn to do this by paying attention to the shape of objects and by disregarding attributes such as color, texture, or function.

You can encourage students to pay attention to the shape of objects by asking them to sort concrete objects in terms of their shape.

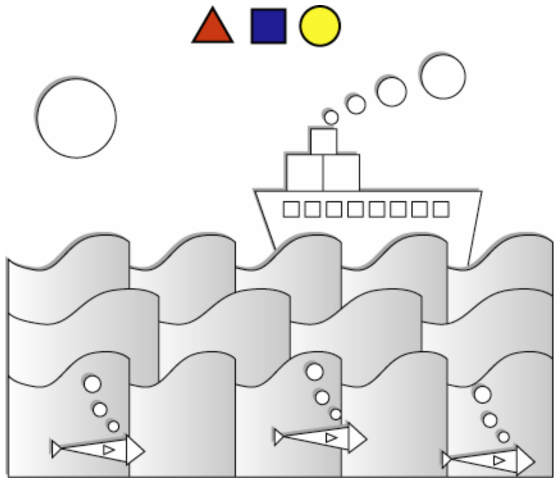
Alternatively, you can show some shapes and have students identify every-day objects that are of that shape. Or, given an every-day object, ask students to identify some shapes. For example, given a circle, students might identify a clock. You can also ask students to imagine or visualize every-day objects that have a circle, square, or rectangle as a feature.

In grade 1, it is important that students have a lot of experience in composing and decomposing various shapes. When students put shapes together or take them apart they learn about the geometric characteristics of the shapes. For example, they gain experience in seeing how some shapes fit together without leaving a space (i.e., tessellate) and in seeing how other shapes do not tessellate.

It is not necessary to ask students to describe their work with shapes in words at this stage. It is important however that students have substantial experience in abstracting shape from other characteristics of objects. Here is an example:

Shapes

Find the shapes in the picture and color them

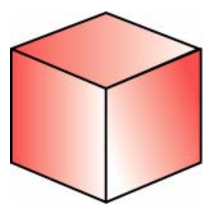


Three-Dimensional Geometric Figures

The second stem relates to three-dimensional geometric figures. In grade 1, students will be expected to recognize three-dimensional shapes in the environment, identify their components, and begin to recognize the difference between two and three dimensional figures.

Here is an example:

How Many Corners Does This Cube Have?

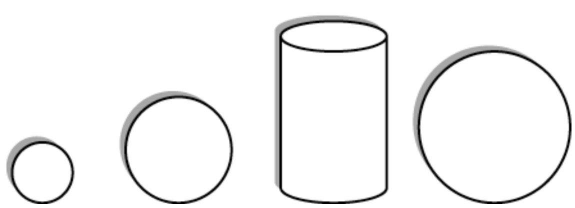


Here is another example:

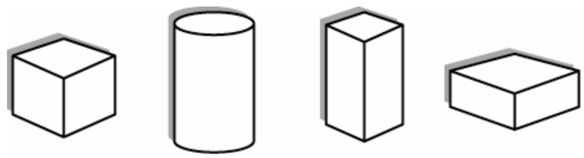
Circle the Odd One Out in Each Group

(a)

(b)



(c)



Congruence

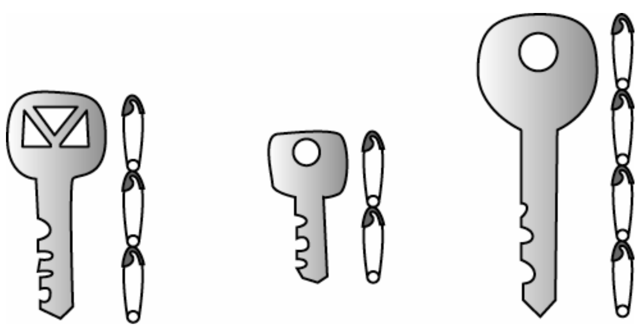
The third stem relates to congruence. As in kindergarten, work with similarity and congruence should take the form of identifying figures that are exactly the same shape. (For very young students, same shape will probably mean same size – and it would probably be unrealistic for these students to comprehend the idea that something might be the same shape but a different size.) During grade 1, students can begin to create congruent figures by drawing figures that have line symmetry or by creating figures that are mirror images of each other.

Geometric Measurement

The fourth stem relates to geometric measurement. In grade 1, students will be expected to use non standard units to measure the height or length of an object.

Here is an example:

Mrs. Sanchez's Three Keys

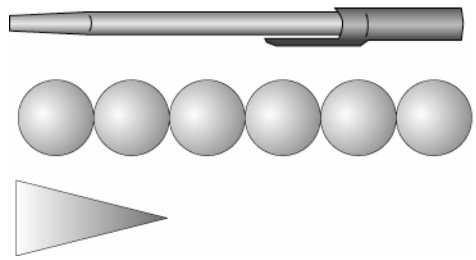


About how many pins long is the longest key?

Here is another example:

This Pen Is About 6 Circles Long.

About how many triangles long is the pen?



Measurement

The fifth stem relates to measurement. In grade 1, students continue to grapple with measuring length and capacity. Just as in Kindergarten, students in first grade must reinforce the idea that a number without a unit (either standard or non-standard) is not a measurement.

When teaching how to measure length the following four stages are useful and widely used by teachers: However, it is unlikely that students will conquer each of these stages before the end of first grade. It seems reasonable, though, for student in grade 1 to have accomplished the first three stages before graduating to grade 2.

Stage One: Direct Comparison

For example, comparing the length of two pencils. Students are given a pair of pencils and asked to decide which pencil is longer or shorter. The students measure the pencils by direct comparison; that is, they align the ends of each pencil and decide which pencil is longer or shorter.

Stage Two: Indirect Comparison

Suppose that you want to compare the length of two objects that cannot be compared directly, you can use the length of some other thing that can be compared to both objects directly.

For example, suppose you want to compare the adjoining two sides of a rectangular table. Put an unmarked and disposable tape to one of the sides that you want to compare and cut the exact length of the side. Then place the cut length of tape to the other side of the table for comparison. This will allow you to decide which is longer.

Stage Three: Measurement By Using Non-Standard Units

For example to represent the length of a side of a table, take a post card or index card, and count the number of cards needed to represent the length of the table. The non-standard unit is card length.

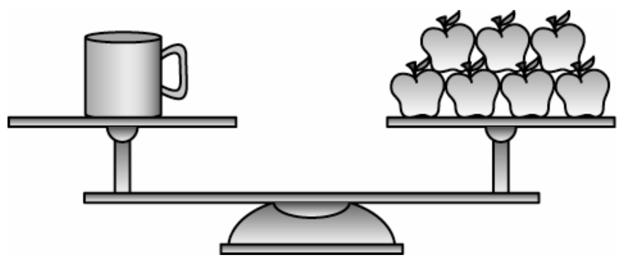
Stage Four: Measurement By Using Standard Units

Measure the length of side of a table using a ruler calibrated in inches or centimeters.

These four stages are designed to enable students to understand the principles of measuring length.

Here is an example of an activity that grade 1 students can do in order to help them understand the notion of unit as central to the notion of measurement.

Use  as 1 unit.



The weight of the cup is ____________ units.

Here is another example:

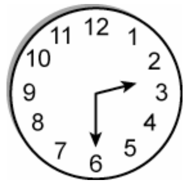
Which one of the two containers below holds more?

Time

The sixth stem relates to time. During grade 1, students must be given the opportunity to determine elapsed and accrued time as it relates to calendar patterns (i.e., days of the week or months of the year). Students should be able to sequence events in a day and recognize time to the hour and to the one-half hour. Here is an example:

Tell the Time

Match the clock to the time



10:00

5:00

10:30

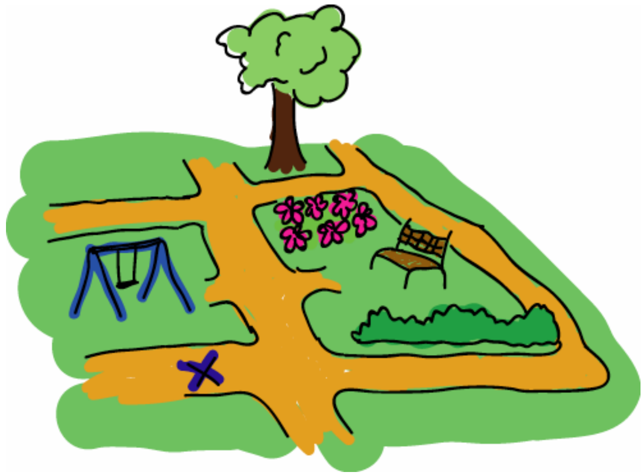
2:30

Spatial Relationships

The seventh stem relates to spatial relationships. In grade 1, students will be expected to describe location and position by using words such as *close by*, *on the right, underneath, above,* and *beyond* to describe where one object is in relation to another or to give directions.

Here is an example:

> From the spot marked X give directions to the following places using words such as: left, right, straight ahead.
>
> The tree.
>
> The flower garden.
>
> The swing.



Grade 2

At the grade 2 level, the Grade Level Expectations specify five stems that address (1) two-dimensional geometric figures, (2) congruence, (3) geometric measurement, (4) measurement, and (5) spatial relationships.

Two-Dimensional Geometric Figures

The first stem relates to two-dimensional geometric figures. In grade 2, as in earlier grades, the emphasis must be on concrete manipulation. Therefore, it is important to ensure that students are given many opportunities to classify shapes as well as compose or decompose them.

In grade 2, you should begin to focus on the characteristics of triangles and quadrilaterals. In doing so, your students will learn about vertices and sides of triangles and quadrilaterals, as well as how to identify and count these components. For students to grasp these ideas their work needs to be concrete. That is, your students should have the opportunity to pay attention to the sides and vertices of triangles and quadrilaterals through hand-on activities that involve classifying, decomposing, and composing.

Your students' work with quadrilaterals can be made more real by investigating the components of boxes (i.e., cubes and rectangular prisms). Many young children are fascinated by boxes and so this provides a familiar context within which to extend the work of identifying and counting vertices, edges and faces.

As your students sort, cut, and build, it is essential that their efforts are guided toward specific fundamental ideas of geometric figures. For example, you could draw your students' attention to a square by asking them to cut a rectangle so as to make one or more squares. Or you could ask students to use congruent squares to create rectangles of different sizes.

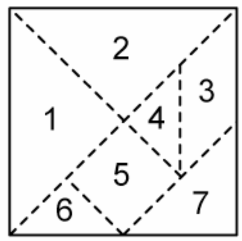
Similarly rectangles (including squares) can be cut or decomposed so as to produce triangles. You can also help children to see that a rectangle can be cut along its diagonal to make two congruent right triangles. In addition, right triangles (and other triangles) can be produced by cutting rectangles in various other ways besides along a diagonal. It is important that you give students the opportunity to explore these ideas.

Specific work with squares and rectangles is important at grade 2 because it can support students' conceptual understanding of congruency and similarity—two other important aspects of the *Geometry and Measurement Strand.*

Here is an example of a grade 2 activity:

Making Pictures

Draw and cut up a square as shown in the picture. Use these 7 figures to create a picture.



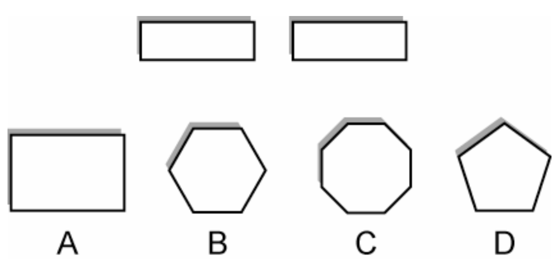
Congruence and Similarity

The second stem relates to congruence. In grade 2, students should be given the opportunity to develop the understanding that two objects are congruent if they are exactly alike except for their position in space. Stacks of congruent objects are familiar to students, especially as seen in a grocery store or the supply room. Also tiles, quilt patterns, and windows often represent congruent shapes. Young students should be given the opportunity to recognize congruent figures in the world around them. This exercise can be strengthened by including examples of figures that are somewhat alike, but which are *not* congruent. In addition, students might use pattern blocks to replicate a given figure and so create an example of a congruent figure, or use line symmetry to decompose a drawing into two congruent parts.

Congruent Rectangles

Which one of the following shapes can be made from the two congruent rectangles?

Two congruent rectangles



Geometric Measurement

The third stem relates to geometric measurement. In grade 2, students should be given the opportunity to calculate distance around (perimeter) and area by surrounding and covering polygons. Here, students begin to grapple with the idea that most measurements are approximations and that repeated measurements reveal this variability. Of course at this grade the idea that a number without a unit is not a measurement must be stressed. Although students have started to use standard units by this grade, it is important for them to understand that a number accompanied by a nonstandard unit is indeed a valid measurement. Here is an example:

Trim

Ms Brown wants to put a trim around this rectangle. How much trim does she need?

Measurement

The fourth stem relates to measurement. In grade 2, students should begin to use standard units to measure length and distance and be able to covert one unit to another. At grade 2, such conversions are always made within systems (i.e., from centimeters to meters or from inches to feet). The metric system provides a perfect system of measurement units for students in grade 2 because it corresponds with the base ten nature of the number system that students are learning. Thus, conversions within the metric system involve multiples of 10 or 100 which is much more accessible than, say, conversions within the English system of measurement (e.g., to covert from feet to inches students must multiply by 12).

Spatial Relationships

The fifth stem relates to spatial relationships. During grade 2, students should be given the opportunity to demonstrate an understanding of spatial relationships using positional language in two- and three-dimensional situations. They should be able to describe and interpret relative positions such as “above the surface of the desk” and “below the triangle on the paper.” Students should also be able to create and interpret simple maps and name locations on coordinate grids.

Grade 3

At the grade 3 level, the Grade Level Expectations specify seven stems that address (1) two-dimensional geometric figures, (2) congruence, (3) similarity, (4) geometric measurement, (5) measurement, (6) spatial relationships, and (7) spatial visualization.

Two-Dimensional Figures

The first stem relates to two-dimensional geometric figures. In grade 3, students extend their work with vertices and sides and use this to describe, identify, and distinguish among triangles, squares, rectangles, rhombi, trapezoids, hexagons, or circles.

In this grade it is important for students to recognize squares, rectangles, rhombi, and trapezoids as special quadrilaterals:

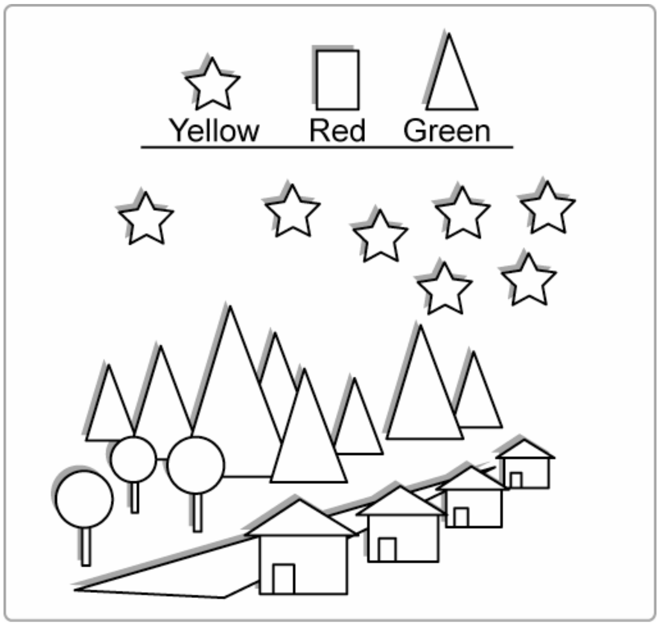
> A rectangle is a quadrilateral with four right angles.
> A square is a rectangle with four equal sides.
> A rhombus is a quadrilateral with four sides of equal length, and
> A trapezoid is a quadrilateral with at least one pair of parallel sides.

Many textbooks define a trapezoid as having exactly one pair of parallel sides. Many others define it as it is here as having at least one pair of parallel sides. This definition is used more widely than the other and is the one preferred by mathematicians.

At this grade, give students the opportunity to classify these special quadrilaterals. For example, very young students can entertain the notion that a square is a rectangle and a rhombus. They can also entertain the notion that a rectangle or a rhombus is not necessarily a square. At this grade it is important that students understand that a hexagon or a triangle is not a quadrilateral.

As in earlier grades student work must be concrete. For example, encourage students to see if it is possible to build triangles, squares, rectangles, rhombi, trapezoids, hexagons or circles from a set of triangles, squares, rectangles, rhombi, trapezoids, hexagons, or circles. Students will see that trapezoids can be built from squares and triangles, but circles cannot be built from any set of polygons. Here is an example:

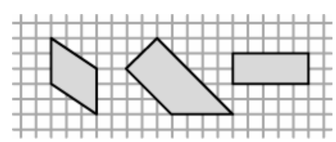
Find the Shapes in the Picture and Color Them.



Here is another example:

Making Shapes

Copy the quadrilaterals shown here three times.
In each, draw a line segment to form:



- One triangle and one quadrilateral;
- Two triangles;
- Two quadrilaterals.

Here is yet another example:

Sides of a Rectangle:

One side of a rectangle is 5 feet long. Another side of the rectangle is 7 feet long. What are the lengths of the other 2 sides of the rectangle?

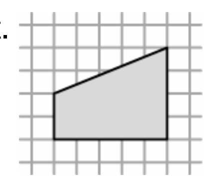
Congruence

The second stem relates to congruence. In order to extend their understanding of congruence, students need extended hands-on experience with rotations, reflections, and translations. This experience can come from actually rotating, reflecting, and translating pattern blocks. Experience with rotations, reflections, and translations can also come from working with drawings and tracing paper or other transparent paper. In grade 3, students should also continue to create congruent figures through composing and decomposing two- and three-dimensional figures and through drawing or cutting with scissors across lines of symmetry.

Here is an example of an activity that students could work on in grade 3.

Two Congruent Quadrilaterals

Draw two quadrilaterals like this one and cut them out.
Put them together to make a square, then put them together to make a pentagon.
What other shapes can you create?



Similarity

The third stem relates to similarity. In grade 3, students should be given the opportunity to develop an intuitive understanding of similar figures. To do this, students will need to develop an understanding of what it means for two figures to differ in size but still be the same shape. For example, students should be shown scale models of objects to help understand that similar means the same shape but not necessarily the same size. These young students can understand that miniature animals and houses are all small versions of familiar animals and objects. The fact that young children know what these tiny objects are supposed to represent shows that they intuitively understand change of scale. Building and taking apart scale models will give students a firm grasp of this idea.

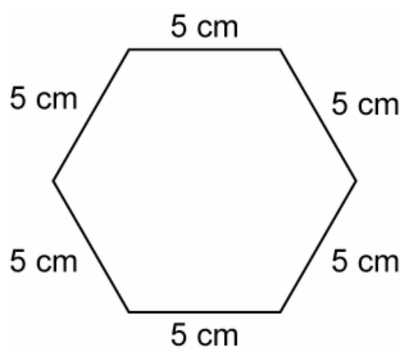
Geometric Measurement

The fourth stem relates to geometric measurement. In grade 3, students should have the opportunity to work with distance around polygons (perimeter) and area of rectangles on grids. Using grids students can count or estimate the square units of area and count or measure the linear units of length.

Here is an example:

Perimeter

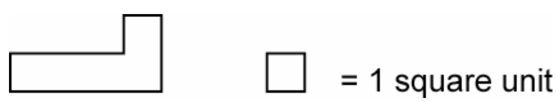
What is the perimeter of this figure?



Here is another example:

Area

What is the area of this figure?



Measurement

The fifth stem relates to measurement. The important issues involved in measurement must never be lost. That is, to be a measurement a number needs to be accompanied by a unit and most measurements are approximations. In addition, students should have the opportunity to learn that the accuracy of a measurement depends to a large extent on the choice of unit. Therefore, students should be given the opportunity to select the unit used in assignments involving the measurement of length.

Spatial Relationships

The sixth stem relates to spatial relationships. During grade 3, students should be able to demonstrate understanding of spatial relationships by interpreting and giving directions from one location to another using positional words. Students should also be able to describe locations on a map or coordinate grid (first quadrant) using positional words or compass directions.

Spatial Vizualization

The seventh stem relates to spatial visualization. In grade 3, students will be expected to represent two-and three-dimensional figures by drawing and building models. If students were asked to create a model of a three-dimensional figure in this grade they would always be given the actual figure to work from. Here is an example that cuts across the first and seventh stem at this grade:

Drawing Shapes

Read the information in the box below.

A polygon with 6 sides.

Fill in the blank:

This shape is a ______________.

Draw this shape.

Grade 4

At the grade 4 level, the Grade Level Expectations specify eight stems that address (1) two-dimensional geometric figures, (2) three-dimensional geometric figures, (3) congruence, (4) similarity, (5) geometric measurement, (6) measurement, (7) spatial relationships, and (8) spatial visualization.

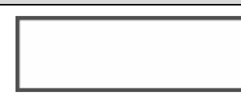
Two-Dimensional Geometric Figures

The first stem relates to two-dimensional geometric figures. In grade 4, students will focus more formally on the ideas of perpendicularity and parallelism. It is better that students learn about the ideas of perpendicularity and parallelism in reference to specific geometric figures such as triangles, squares, rectangles, rhombi, trapezoids, hexagons, circles, or octagons. Grounding these ideas in shapes is more concrete than studying lines that are parallel or perpendicular.

At this grade, students can revisit the shapes that they have studied in grade 3 and bring the concepts of perpendicularity and parallelism to bear. For example, it is important that students see that a rectangle, a square, and a rhombus all have two pairs of parallel sides. This will help students understand what is meant by parallel as they will see that, for these shapes, opposite sides are always the same distance apart. Here is an example:

Shapes

Complete the table below by writing the number of sides and sketching each polygon. The first one has been done for you.

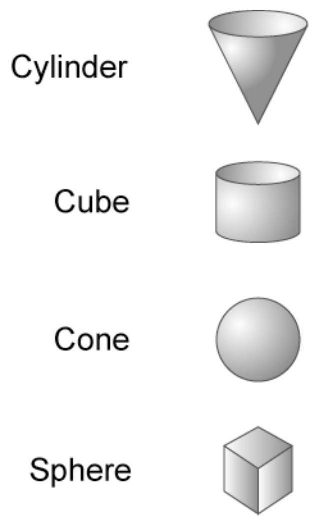
Shape	Number of Sides	Sketch of Shape
Rectangle	4	
Triangle		
Trapezoid		
Hexagon		
Rhombus		

Three-Dimensional Geometric Figures

The second stem relates to three-dimensional geometric figures. At grade 4, it is important that students continue to classify rectangular and triangular prisms, cylinders, or spheres. For example students could classify these shapes by the shape of their bases, or by the existence or absence of what might be called a base. Here is an example:

Shape and Name

Match the picture of each shape to its name.

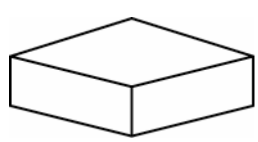


Congruence

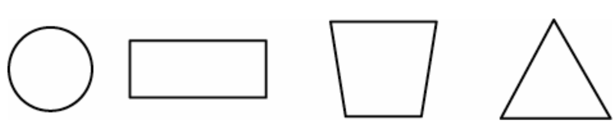
The third stem relates to congruence. In grade 4, students need to understand that rotation, translation, and reflection move a figure in the plane from one position to another without changing the lengths of its sides or its angle measurements. In this grade, students should also continue to build congruent figures using line symmetry and pattern blocks. Here is an example:

Congruent Face

Look at the object below.



Put a circle around the shape below that could be congruent to one face of the block above.



Here is an example related to lines of symmetry:

Lines of Symmetry

Look at the shape below.

How many lines of symmetry does this shape have?

Draw the lines of symmetry on the shape.

Similarity

The fourth stem relates to similarity. In grade 4, students should be given the opportunity to create simple maps and drawings to scale, and to understand that scale drawings produce figures that are the same shape, regardless of size. Students should always be asked to complete their scale drawings and maps on rectangular grid paper. Students need to understand that similar means “same shape” but not necessarily “same size.” At this grade level, students should be asked to identify similar figures and to use the ideas of similarity to solve problems. For example, students could be shown squares (or equilateral triangles) that are not necessarily the same size and then be asked if the squares are the same shape. Then students could compare sets of rectangles that are the same shape with sets of rectangles that are not the same shape. At this grade level, students should also be given the opportunity to build and take apart scale models so as to deepen their grasp of the idea of similarity.

Geometric Measurement

The fifth stem relates to geometric measurement. In grade 4, students will continue to work with the distance around polygons (perimeter) and the area of rectangles. In this grade, you should give students the opportunity to find the area of polygons and other composite and irregular shapes. In grade 4, students should be encouraged to develop formulas for the distance around and for area. Through working with formulas students will begin to realize that area measurement is based on measurement of length. This means that the measurement of area is a *derived* measurement, or, a measurement that can be derived by multiplying together two linear measurements. Another way to think of this is as follows: you can measure length directly by using a tool such as a ruler. However, no such similar tool exists for the measurement of area. Instead the ruler must be used to generate the lengths and then these lengths must be used to derive the area measurement.

Measurement

The sixth stem relates to measurement. When students are involved in measuring a quantity, it is often useful for them to be able to estimate measurements. Estimating length is fundamental and is the basis of estimation of area or volume. That is, if one can estimate the linear dimensions of an object, then these estimations can be used to compute area and volume. Therefore, it is recommended that the teaching of measurement is accompanied with the teaching of estimation and approximation. To be able to estimate measurements, students need to develop a robust sense of quantity.

Students need to become familiar with common measurement units such as one inch, one centimeter, one foot, one yard, one meter, one pound, and one kilogram.

In order to hone their approximation skills, students should develop different ways of measuring common objects. For example, students could estimate the approximate height of a school building by estimating how many times higher the building is than a person of known height standing next to it. Or they could use the approximate distance walked in 1 minute to measure the distance between two points. Finally, they could estimate the approximate length of a building by counting the number of hand-spans of a known length that are needed to cover it.

With a strong sense of quantities and units, and with the ability to estimate, one can chose appropriate units and instruments to solve measurement problems. Students need to learn that all measurements of length are approximations. Measurement always involves some form of error. Therefore, it is important to learn to use tools properly and to measure several times in order to decrease this error. In later grades students will learn to take the average in order to find a more accurate measurement. Here is an example:

Which Units?

Which of the following units of measure is *best* for measuring the length of this line?

(a) feet

(b) yards

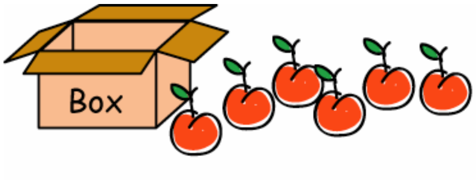
(c) inches

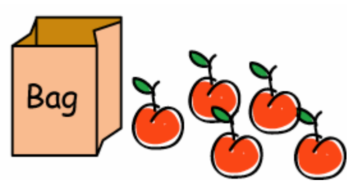
(d) pounds

Here is another example:

Larger Container

Which one of the two containers below holds more?

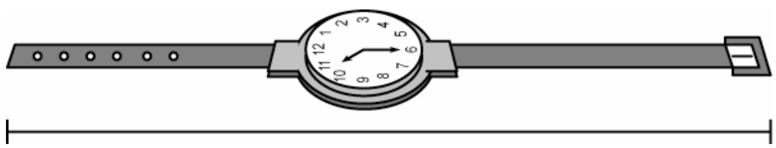




Throughout elementary school it is advisable to continue to give students plenty of practice using a ruler to measure the length of common objects: Here is an example:

Length of a watch

Use your ruler to help you solve this problem. How many inches long is the watch shown below?



Spatial Relationships

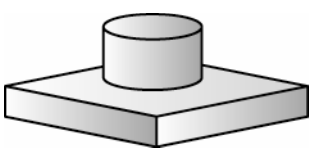
The seventh stem relates to spatial relationships. During grade 4, students should be given the opportunity to demonstrate understanding of spatial relationships by interpreting and giving directions between locations on a map or the first quadrant of a coordinate grid. Students should also be able to plot points in the first quadrant and find the horizontal and vertical distances between points on a coordinate grid in the first quadrant.

Spatial Vizualization

The eighth stem relates to spatial visualization. In grade 4, students will be expected to represent two-and three-dimensional figures by drawing and building models. In this grade students could be asked to create a model of a rectangular prism given a two-dimensional representation of one. Here is an example of what a student in grade 4 might be expected to do:

What Shapes?

What shapes make up this geometric figure?



Grade 5

At the grade 5 level, the Grade Level Expectations specify eight stems that address (1) two-dimensional geometric figures, (2) three-dimensional geometric figures, (3) congruence, (4) similarity, (5) geometric measurement, (6) measurement, (7) spatial relationships, and (8) spatial visualization.

Two-Dimensional Geometric Figures

The first stem relates to two-dimensional geometric figures. In grade 5, students will be expected to identify, describe, or classify triangles and quadrilaterals. At this grade students should have experience classifying triangles according to angle size (i.e., acute, right, obtuse). In addition, students will need to be able to recognize an equilateral or an equiangular triangle. In this work it is important for students to reason that it is not possible to have a equilateral triangle that is also a right or obtuse triangle. This type of reasoning will help reinforce the idea in geometric measurement that the sum of the angles of a triangle is 180 degrees.

Through their work classifying quadrilaterals, students will realize that a square, a rectangle, or a rhombus is also a parallelogram.

In addition, if students learn that a trapezoid is a quadrilateral with at least one pair of parallel sides, they can begin to understand that all parallelograms are, in fact, trapezoids. It is important to note that two different definitions of a trapezoid exist side-by-side. A quadrilateral is often defined as having just one pair of parallel sides. The definition that a trapezoid is a quadrilateral with at least one pair of parallel sides is the one that is preferred by mathematicians.

Here is an example of a task that relates to two-dimensional figures:

Complete the table below by writing the number of sides and angles, and sketching each polygon. The first one has been done for you.

Shape	Number of Sides	Number of Angles	Sketch of Shape
Rectangle	4	4	
Pentagon			
Hexagon			
Quadrilateral			
Triangle			
Octagon			

Three-Dimensional Geometric Figures

The second stem relates to three-dimensional geometric figures. In grade 5, students extend their work with three-dimensional shapes to include pyramids and cones. Now, students can classify shapes according to the shape of their base, whether or not a base exists, or whether or not the shape comes to a point. Here is an example:

Three-dimensional shapes

Fill in the box next to each item with the *correct number*.

A rectangular prism always has:

☐ faces

☐ edges

☐ vertices

Congruence

The third stem relates to congruence. In grade 5, students should be given the opportunity to use rotation, reflection, and translation to solve problems involving the coordinate plane.

Similarity

The fourth stem relates to similarity. In grade 5, students should be given the opportunity to create scale drawings of triangles and rectangles and to understand that scale drawings produce figures that are the same shape regardless of size. In a scale drawing of a figure, the corresponding angles of the figures are the same and the ratios of the lengths of corresponding sides are also the same. Students should always be asked to complete their scale drawings on rectangular grid paper. Students can also integrate their understanding of congruence and similarity by understanding that when the scale factor of an enlargement is 1, the figures are congruent as well as similar.

Geometric Measurement

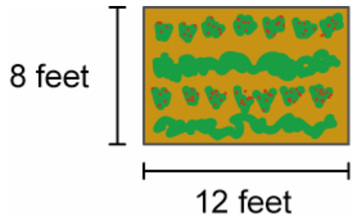
The fifth stem relates to geometric measurement. In grade 5, students will continue to work with the distance around polygons (perimeter) and the areas of rectangles, polygons, and composite and irregular shapes. In this grade, you should give students the opportunity to use the area of a rectangle to deduce the area of a right triangle. As in grade 4, grade 5 students should be encouraged to develop formulas for distance around and for area.

In grade 5, students take their work with three-dimensional figures to a new level. Here they are to find the volume of rectangular prisms. Students should have the opportunity to see that a rectangular prism whose length, width, and height are equal, is a cube. Therefore, a cube is a special rectangular prism, just as a square is a special rectangle. Again, through working with formulas students will begin to realize that volume, like area, is a derived measurement. This means that the measurement of volume is derived from the measurement of length, width, and height. Thus, volume is a measurement that can be derived by multiplying three linear measurements.

Volume and area measurements, as with length, must be given with an appropriate unit since a number without a unit is not a valid measurement. Measurements of volume and area are also approximations since these are based on measurements of length, itself an approximation. Here is an example:

Vegetable Garden

A vegetable garden is shaped like a rectangle, 12 feet long and 8 feet wide.



How many feet of fencing is needed to fence the perimeter of this vegetable garden?

Measurement

The sixth stem relates to measurement. The ideas that most measurements are approximations and that all measurements must be given in terms of a unit are still important ideas at grade 5. For these ideas to be understood, students should be given the opportunity to measure objects. When measuring objects students should be asked to select the most appropriate tools and units. At this grade students are in a better position to understand the idea that most measurements are approximations and that repeated measurements reveal this variability. Here is an example:

Dimensions of Your Desk

Use a ruler to measure your desk to the nearest $\frac{1}{2}$ inch.

Spatial Relationships

The seventh stem relates to spatial relationships. During grade 5, students should be given the opportunity to demonstrate understanding of spatial relationships using location and position by interpreting and giving directions between locations on a map or coordinate grid. Students should also be able to plot points in four quadrants and identify the vertices of polygons as they are reflected, rotated, and translated in the first quadrant of a coordinate grid and determine horizontal and vertical distances between points on a coordinate grid in the first quadrant.

Spatial Vizualization

The eighth stem relates to spatial visualization. In grade 5, students will be expected to represent two- and three-dimensional figures by drawing and building models. In this grade students could be asked to create a model of rectangular prisms, cones, cylinders, or pyramids given two- or three-dimensional representations.

Grade 6

At the grade 6 level, the Grade Level Expectations specify six stems that address (1) two-dimensional geometric figures, (2) three-dimensional geometric figures, (3) congruence, (4) similarity, (5) geometric measurement, and (6) measurement.

Two-Dimensional Figures

The first stem relates to two-dimensional geometric figures. In grade 6, students work classifying triangles can reach a new level because they are expected to be able to classify triangles by angle (i.e., acute, obtuse, right) and by side (equilateral, isosceles, and scalene).

To do this work, students must pay attention to the length of the sides of triangles. It is here that we want students to learn that an isosceles triangle has two sides of equal length and an equilateral triangle has three sides of equal length. It is appropriate for students to learn that an equilateral triangle is an example of a special isosceles triangle.

Students can be given an interesting reasoning problem by asking them to complete this two-way classification grid by drawing an example of a triangle that meets both criteria or by explaining why a triangle cannot be drawn:

	Equilateral	Isosceles	Scalene
Acute			
Obtuse			
Right			

Similar classification work can be created for quadrilaterals. For example, students can be asked to classifying quadrilaterals according to the number of right angles and the number of pairs of parallel sides.

Three-Dimensional Geometric Figures

The second stem relates to three-dimensional geometric figures. In grade 6, work with three dimensional figures students pay attention to rectangular and triangular prisms, cylinders, pyramids, cones, or spheres. Students must be given the opportunity to classify these according to the numbers of edges and vertices as well as the shape and number of bases.

Congruence

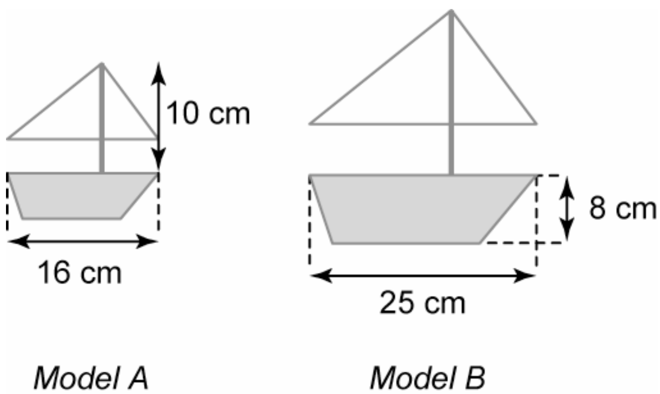
The third stem relates to congruence. Grade 6 students should know that if *any* two polygons can be laid one on top of the other by using rotations, reflections, and translations, these two polygons are congruent. They should understand why rotations, reflections, and translations can be used to generate congruent figures. In addition to line symmetry, students should use rotational symmetry to demonstrate congruent parts of a figure or drawing.

Similarity

The fourth stem relates to similarity. In grade 6, students should be given the opportunity to create scale drawing of polygons and to understand that scale drawings produce figures that are the same shape regardless of size. In a scale drawing of a figure, the corresponding angles of the figures are always the same and the ratios of the lengths of each pair of corresponding sides are the same, between the scale drawing and the full-sized figure. The polygons used as examples for these concepts should be relatively simple, and should be restricted to triangles and quadrilaterals so as to enable students to see these relationships without introducing undue complications. Here is an example:

Scale Models

These are two accurate scale models of the same yacht.



(Not to scale)

Use the measurements given in the diagrams to calculate:

The height of the mast in Model B.

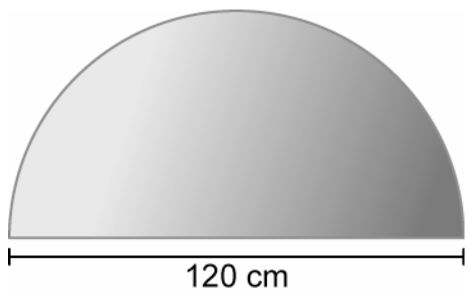
The depth of the hull on Model A.

Geometric Measurement

The fifth stem relates to geometric measurement. In grade 6, students will extend their work with distance around polygons (perimeter) to include circumference. They will also extend their work with area to include finding the area of quadrilaterals and triangles. In grade 6, students use the volume of rectangular prisms to solve problems. Here is an example:

Tables

Jesus and Ana are making tables for a new coffee shop. The table tops are to be shaped like semicircles as shown below.



The top of the table is made of wood and a metal edge is to be fixed to its perimeter.

Calculate the total length of the metal edge.

The coffee shop needs 16 tables. Jesus and Ana have 50 meters of the metal edge in the workshop. Will this be enough for all sixteen tables? Give a reason for your answer.

Measurement

The sixth stem relates to measurement. At this grade, students should be given the opportunity to measure physical objects and to carry out intra-system unit conversions. The ideas that most measurements are approximations and that all measurements must be given in terms of a unit are still important ideas at grade 6. Here is an example of a measurement task:

Price Per Square Inch?

This protective pad is used to keep furniture from scratching the floor.

This pad is $\frac{1}{2}$ inch wide and 72 inches long and is rolled into a coil before being packaged.

The pad costs $4.99.

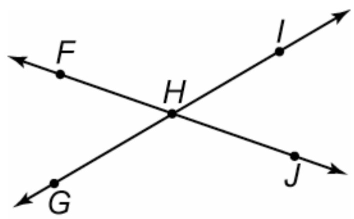
What is the cost of the pad per square inch?

Grade 7

At the grade 7 level, the Grade Level Expectations specify six stems that address (1) two-dimensional geometric figures, (2) geometric theorems, (3) congruence, (4) similarity, (5) geometric measurement, and (6) spatial visualization.

Two-Dimensional Geometric Figures

The first stem relates to two-dimensional geometric figures. In grade 7, students learn about the relationships between angles and lines. For example, students must learn what is meant by vertically opposite angles and learn that vertically opposite angles are congruent: This diagram illustrates what is meant by vertically opposite angles:



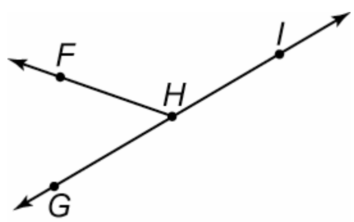
$\angle FHI$ and $\angle GHJ$ are vertically opposite angles.

$\angle FHG$ and $\angle IHJ$ are also vertically opposite angles.

The measures of vertically opposite angles are equal.

Vertically opposite angles are congruent.

Students must also learn what is meant by a "straight angle" or "angles on a line". A linear pair of angles is an example of what is called a straight angle or angles on a line. Students must learn that linear pairs of angles, straight angles, or angles on a line sum to 180 degrees. This diagram illustrates what is meant by angles on a line:



$\angle FHI$ and $\angle FHG$ *is an example of* angles on a line.

Angles on a line add up to 180°.

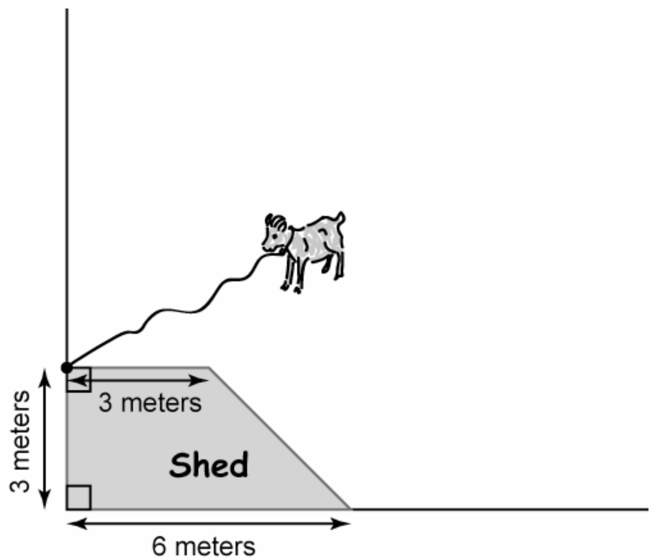
In some parts angles on a line are called "straight angles". Two angles on a line is a linear pair of angles.

Students must also learn what is meant by adjacent angles. Adjacent angles are two or more angles in a plane which share a common vertex and a common side but do not overlap. Incidentally, the diagram above also illustrates adjacent angles, because $\angle FHI$ and $\angle FHG$ *are adjacent angles.* You can see that $\angle FHI$ and $\angle FHG$ share a common vertex and a common side but do not overlap. Of course, adjacent angles do not need to add to 180 degrees. When two adjacent angles add to 180 degrees they are called a *linear pair* of angles.

In addition to these, grade 7 students must learn the angle relationships that emerge when two parallel lines are cut by a transversal. The mathematics of the following example combines the mathematics of this stem and the stem that relates to geometric measurement:

Grazing Goat

Below is a scale plan of a goat G tethered by a 5-meter rope to a shed in the corner of a large field. The goat cannot go inside the shed.



Draw and shade the region where the goat can graze.

Find the area of this region.

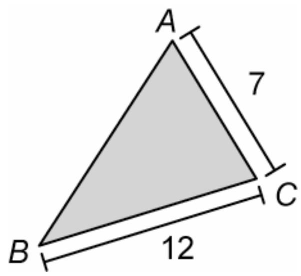
Geometric Theorems

The second stem relates to geometric theorems. In particular the GLEs specify that students must learn to apply the triangle inequality theorem. In this grade students must also know and use the relationship between the sum of the interior angles of a polygon and a count of its sides. Students must be able to calculate the sum of the interior angles of a regular polygon. Many students will learn this by first seeing that a polygon with n sides can be decomposed into $(n – 2)$ triangles. Therefore, the sum of the interior angles of a polygon with n sides can be calculated using the formula $(n – 2)180$.

In grade 7 students must use these relationships or theorems to solve problems. Here is an example:

Possible Triangles

For triangle *ABC* shown below, what values of *AB* are not possible?



Congruence

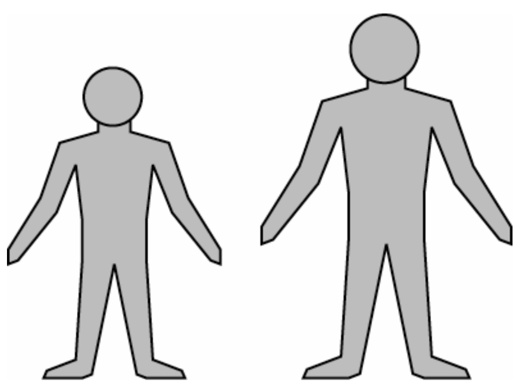
The third stem relates to congruence. In grade 7, students should be given the opportunity to learn that when *any* two figures (not just polygons) on a plane can be laid one on top of the other by rotations, reflections, and translations, these two figures are congruent. Grade 7 students should also use rotations, reflections, and translations on a coordinate plane to solve problems involving the congruence of polygons.

Similarity

The fourth stem relates to similarity. In grade 7, students should be given the opportunity to learn that any two similar polygons with scale factor r have areas related by a factor of r^2. Grade 7 students should also be given the opportunity to use this understanding to solve problems involving similar figures, linear measures and area. Here is an example:

Father and Son

This is a scale drawing of a man and his son. The actual height of the man is 1.81 meters.



Use the measurement given to find the son's height.

Geometric Measurement

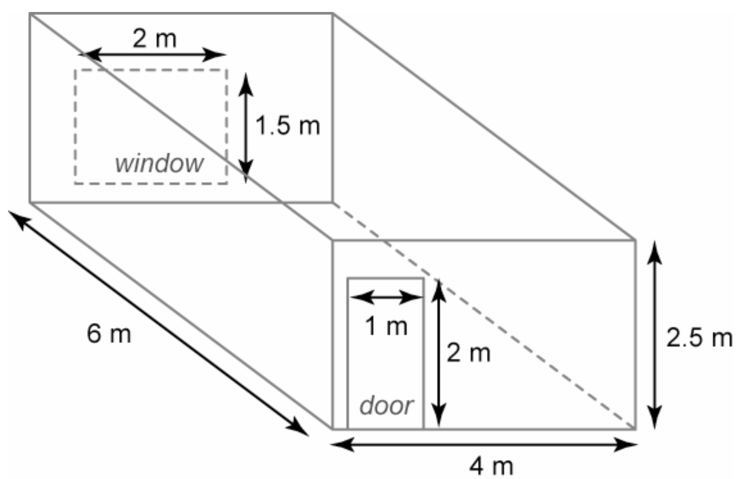
The fifth stem relates to geometric measurement. In grade 7, students will extend their work with distance around plane figures to include figures whose perimeter is composed of line segments and parts of circles. They will also extend their work with area to include finding the area of composite figures such as those that can be decomposed into quadrilaterals, triangles, and circles or parts of circles and the surface area of rectangular prisms. In grade 7, students extend their work with volume by finding the volume of triangular prisms and cylinders.

At this grade students should be given the opportunity to measure physical objects. The ideas that most measurements are approximations and that all measurements must be given in terms of a unit are still important ideas in grade 7. Here is an example:

Painting a Room

Shawn is planning to paint the walls of her room

The room is in the shape of a rectangular prism with the dimensions shown.



One liter of paint will cover approximately 8.58 square meters. How much paint does Shawn need to paint the walls of her room?

The paint she wants is sold only in 1 liter and 2.5-liter cans.

2.5 liters costs $12.75

1 liter costs $6.50

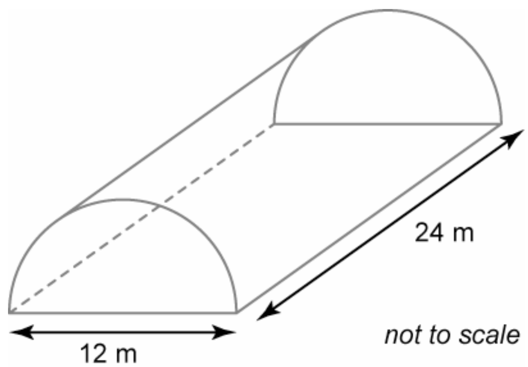
What will be the minimum cost of painting Shawn’s room?

Spatial Visualization

The sixth stem relates to spatial visualization. In grade 7, students must learn to represent three-dimensional figures in two dimensions. At this grade they will be expected to do this by sketching or making nets of triangular prisms, cylinders, and pyramids. Students will also be expected to use the nets of three-dimensional figures as a tool for finding surface area. Here is an example:

Storage Tunnel

A diagram of a storage tunnel is shown below. The curved surface of the tunnel is made from tough fabric and the semi-circular sides and the rectangular base is made from wood. Draw a net of the storage tunnel. About how much fabric and wood is needed to make a storage tunnel? What is the volume of the storage tunnel?



Grade 8

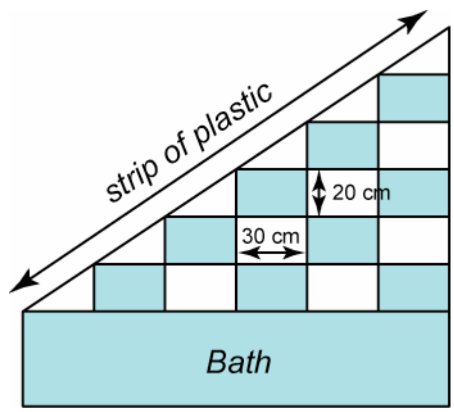
At the grade 8 level, the Grade Level Expectations specify three stems that address (1) geometric theorems, (2) similarity, and (3) geometric measurement.

Geometric Theorems

The first stem relates to geometric theorems. In grade 8, students learn about the Pythagorean theorem, and they learn to use it to solve problems and to find the missing side of a right triangle. Here is an example that students should be able to tackle in grade 8:

Tiling the Bathroom

The diagram below shows the wall Jamie has tiled above the bath in his bathroom. He used rectangular tiles, some of which he cut in half. The length of each tile is 30 centimeters. The width of each tile is 20 centimeters. A strip of plastic is fitted along the top of the tiles. What length of the strip of plastic is needed?



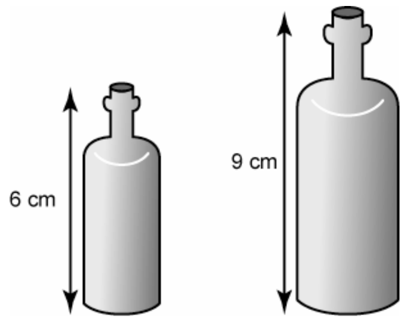
Similarity

The second stem relates to similarity. In grade 8, students need to be given the opportunity to learn that any two similar solids with scale factor r have surface areas related by a factor of r^2 and volume related by scale factor r^3. Grade 8 students also need to be given the opportunity to use this understanding to solve problems involving similar figures, linear measures, lateral area, and volume. Students should understand that when $r = 1$ the figures are congruent.

Here is an example that students should be able to tackle in grade 8:

Similar Bottles

Two bottles are mathematically similar. The smaller one is 6 centimeters high and holds 30 milliliters of liquid. The larger one is 9 centimeters high. What is the volume of the larger one?



Geometric Measurement

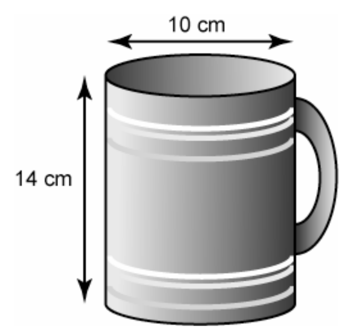
The third stem relates to geometric measurement. In grade 8, students will extend their work with surface area and volume by finding the volume and surface area of cylinders, pyramids, or cones.

At this grade students should be given the opportunity to measure physical objects. The ideas that most measurements are approximations and that all measurements must be given in terms of a unit are still important ideas at grade 8. Here is an example that students should be able to tackle in grade 8:

Coffee Mug

What is the volume of the mug shown opposite?

Suppose that 600 milliliters of coffee are poured in to the mug, what is the depth of the coffee in the cup.



Grade span 9-10

At the 9-10 Grade Span, the Grade Level Expectations specify seven stems that address (1) proof, (2) congruence, (3) similarity, (4) geometric measurement, (5) measurement, (6) coordinate geometry, and (7) spatial reasoning.

Proof

The first stem relates to proof. Throughout Grades 9-10, students create formal proofs about angles, perpendicular and parallel lines, midpoints, circles, and polygons.

Students need to be able to know and use various ideas associated with angles to construct mathematical arguments. For example, students need to know what is meant by vertically opposite angles, straight angles and a linear pair of angles. They need to know that when the measures of angles sum to 90 degrees the angles are called *complementary* angles and when the measures of angles sum to 180 degrees the angles are called *supplementary* angles. Also, students need to know what is meant by the term, *angle bisector*.

Students need to know and be able to use various ideas associated with lines to construct mathematical arguments. For example, they need to know what is meant by perpendicular, parallel ,and skew lines. They also need to know how to find the mid-point of a line and use it solve problems.

At these grade levels, students need to know and be able to use various ideas associated with circles to construct mathematical arguments. For example, what is meant by the terms radius, diameter, circumference, center, chord, arc, sector, and segment of a circle. Students will need to know what is meant by angles subtended at the centre and angles subtended at the circumference. Students will also need to know and be able to prove theorems about central angles and angles at the circumference. They should be able to investigate the directly proportional relationships between arc length and the measure of its associated central angle, as well.

Students will need to be able to prove the Pythagorean theorem and prove the Triangle Inequality Theorem as well as use the basic trigonometric ratios (sine, cosine, and tangent).

In the first two years of high school, students are expected to use all of this geometry to solve problems within the discipline of mathematics and in other disciplines such as architecture.

Congruence

The second stem relates to congruence. At these grade levels, students should use rotations, reflections, and translations on a coordinate plane to solve problems. While doing this work, students should also be given the opportunity to learn the formal conditions for congruent triangles (e.g., side-angle-side and side-side-side congruence tests).

Similarity

The third stem relates to similarity. In previous grades, students will have had the opportunity to learn about similarity and similar figures. By the end of grade 8 students will have learned that any two similar solids with scale factor r have surface areas related by a factor of r^2 and volume related by a scale factor of r^3. In this grade students need to be able to demonstrate formally *why* these relationships hold. In grades 9-10 students need to be given the opportunity to use this understanding to solve problems involving volume and surface area of solids, area and length of plane figures, and the height or length of objects.

Students also need to be given the opportunity to learn the definition of a *dilation* (enlargement or contraction) using coordinates and understand its basic properties. In this context, students can understand that a dilation will always move the starting triangle to another similar triangle.

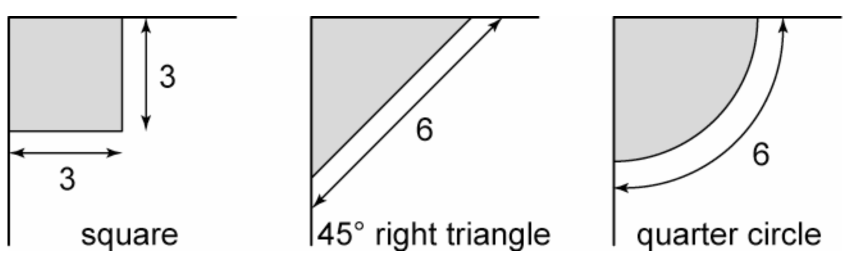
Geometric Measurement

The fourth stem relates to geometric measurement. Students will be expected to extend with length by finding the perimeter of figures composed of lines and parts of circles.

At these grade levels students will also extend their work with the areas of plane figures by finding the area of composite figures that can be decomposed into polygons, circles, or parts of circles. Here is an example of the type of task that students will be expected to complete:

In a Corner

Using intuition, rank the following regions according to their area, largest to smallest.

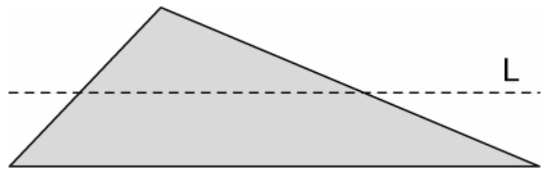


Now calculate the area of each.

Here is another short problem that students will be expected to think about and solve:

Exactly Half

Line *L* is parallel to the base of the triangle. If we want this line to divide the triangle into two equal areas, what should the height of the small triangle be as a proportion of the height of the large triangle?



At these grade levels, students will extend their work with surface area and volume by finding the volume and surface area of solids composed of prisms, cylinders, pyramids, or cones and spheres or hemispheres.

Measurement

The fifth stem relates to measurement. Students will be expected to measure physical objects. The idea that most measurements are approximations and the idea that all measurements must be given in terms of a unit are still important at these grade levels. Here is an example of a measurement problem that students could be asked to solve:

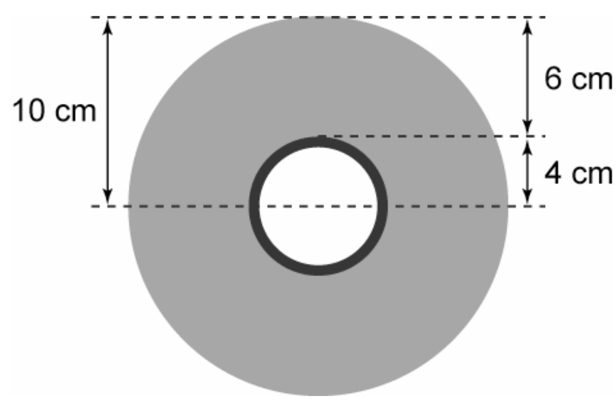
Fertilizer

You have a large load of fertilizer. You assume that the pile is in the shape of a cone, but because of its size you cannot directly measure either its diameter or its height.

Describe how you could estimate the volume of the pile by measuring some of its accessible features.

Roll of Paper

The diagram shows the end view of a roll of paper.
The thickness of the paper in the roll is 0.05 cm.
Figure out the length of the paper.

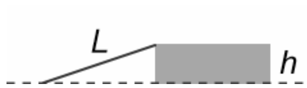


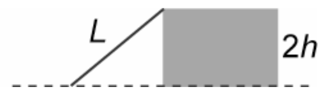
Coordinate Geometry

The sixth stem relates to coordinate geometry. Here, students are expected to solve problems involving the distance formula, midpoints of lines, perpendicular and parallel lines, or slope. Here is an example of a task that students will be expected to solve:

Slope

A board of length *L* is leaning against a platform of height *h*. If the same board is leaned against a platform of height 2*h*, is the slope exactly doubled, more than doubled, or less than doubled? Support your reasoning.

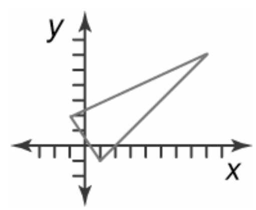




Here is another type of problem that students could be asked to solve:

Is the Triangle an Isosceles?

The sides of the triangle at right are formed by the graphs of $3x + 2y = 1$, $y = x - 2$, and $-4x + 9y = 22$. Is the triangle isosceles? How do you know?



Spatial Vizualization

The seventh stem relates to spatial reasoning. Students are expected to solve problems involving spatial reasoning and visualization.

Here is an example of a spatial visualization task that students should be able to solve:

Locus of points

In each case, sketch the figure consisting of all points, in a plane, that are:

Exactly 2 cm from a given point.

Exactly 2 cm from a given line segment of length 8 cm.

Exactly 2 cm from a given square of side length 8 cm.

Exactly 2 cm from a given circle of diameter 8 cm.

Students are expected to represent three-dimensional objects in two dimensions by providing orthogonal, picture, or isometric views. In these grades, students are expected to use dynamic geometric software to represent three-dimensional figures in two dimensions.

Grade span 11 –12 for all students

At the 11-12 Grade Span, the Grade Level Expectations specify five stems that address (1) proof, (2) geometric figures, (3) similarity, (4) geometric measurement, and (5) spatial reasoning.

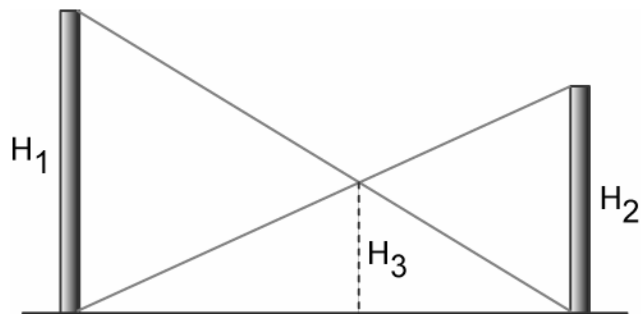
Proof

The first stem relates to proof. At these grade levels students will extend their work with proof by proving propositions about angles, lines, circles, distance, midpoint, congruence, and similarity.

Here is an example of a problem that students could be expected to complete:

Two Poles

Two poles of heights H_1 and H_2 are connected, top to bottom, with straight wires, as shown. It is a fact that the height, H_3, of the crossing point of the wires does not change if the poles were moved closer together or farther apart. (The wires are always straight.) Give a simple argument that proves this fact.



Geometric Figures

The second stem relates to geometric figures. Students should view a circle as the locus of points in a plane which is a fixed distance from a given point and be able to use the distance formula to find the equation of a circle.

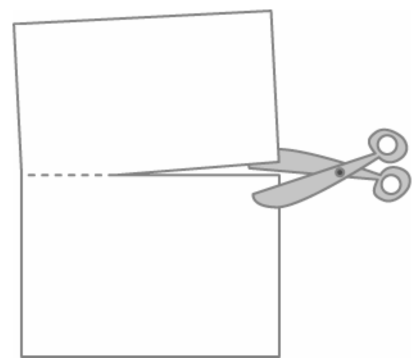
Similarity

The third stem relates to similarity. In grades 11-12, students' work with problems involving similar figures should include both two-dimensional and three-dimensional applications and should be integrated with their study of trigonometry. For example, students should be given the opportunity to solve for the three angles of a triangle after being given the lengths of its sides. Asking students to find the height of a pyramid given basic information about its side lengths and angle measures can extend this type of problem to three dimensions.

During grades 11-12, students should be given the opportunity to understand that similarity of right triangles allows the trigonometric functions to be defined as ratios of right triangles. Students should know and be able to use the ratios of special right triangles to determine the sine, cosine, and tangent of special angles (i.e., angles 30, 45, 60 degrees).

Here is an example of a task that students could be asked to solve:

Shape of Halves



Suppose you cut an 8.5" by 11" piece of paper into two pieces with a cut parallel to the short side. Where would you place this cut so that one of the pieces is similar to (the same shape as) the original piece of paper?
Draw a diagram and label the dimensions.

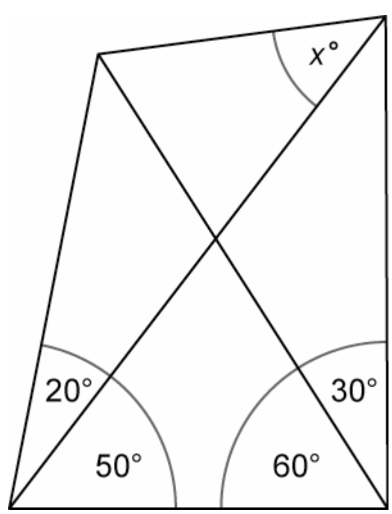
Some rectangular paper has a shape with a special property:
You can cut the paper in half to form two rectangles, each of which has the same shape as the original paper.
If the long dimension of this paper is 11", what is the short dimension?
Draw a diagram and label the dimensions.

Geometric Measurement

The fourth stem relates to geometric measurement. Students will be expected to apply trigonometric formulas to find angles, lengths, and areas of polygons. Here is an example of a challenging geometry problem that students could be asked to grapple with:

Triangles in a Quadrilateral

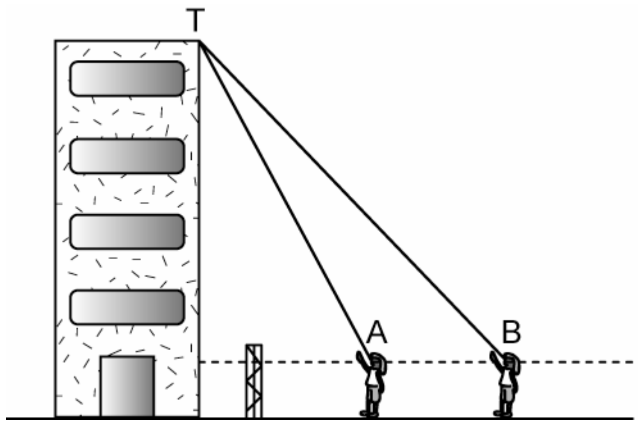
In the following figure, find angle *x*. (All angle measures are in degrees.)



Here is an example of a classic problem that students can be asked to solve:

Height of a Building

The diagram shows two positions (position A and position B) of a student as she views the top of a tower. Position A is just outside a security fence that prevents her from measuring the distance from A to the building.



From position B, the angle of elevation to T at the top of the tower is 64°.

From position A, the angle of elevation to the top of the tower is 69°.

The distance AB is 4.8 meters and the height of the student to eye level is 1.5 meters.

Find the height of the tower.

Spatial Vizualization

The fifth stem relates to spatial reasoning. Students should be able to use a compass and straightedge or dynamic software to form geometric constructions.

Grade Span 11 –12 for Prospective Mathematics Majors

In addition to the mathematics identified in the Grade Span 11-12, students who plan to major in mathematics, engineering, or the sciences need to study additional mathematical concepts.

Proofs

Prospective mathematics majors will be expected to extend and deepen their knowledge of proofs and proof techniques. Here is an example that prospective mathematics majors should be expected to tackle:

> **Two Triangles**
>
> Triangle #1 has sides of length *a*, *b*, and *c*, and angles of measure *A*, *B*, and *C*.
>
> Suppose we are given another triangle, Triangle #2, in which *five out of six* of its side and angle measures are the same as in Triangle #1.
>
> Construct an example of a pair of triangles with this relationship, but that are *not congruent.*

Geometric figures

Specifically, students should be given the opportunity to explore and interpret conic sections both graphically and algebraically, and understand how different planar slices of a double cone yield different conic sections. Students should understand conic sections as loci of points in the plane satisfying certain distance requirements, and be able to use the distance formula to obtain equations for conic sections.

Congruence

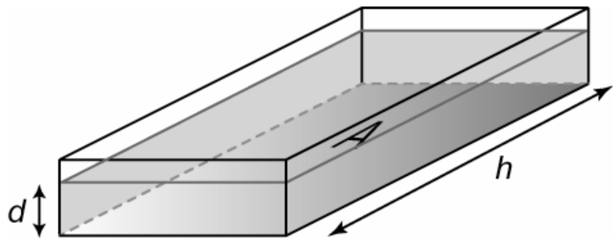
Students should use matrices to represent rigid motion in the plane (i.e., reflection, rotation, and translation).

Geometric Measurement

Students should know Cavalieri's principle and be able to use it to find volumes. Here is an example of a problem that students should be able to solve:

Pouring Liquid

A volume, *V,* in cubic centimeters, of liquid is poured into a rectangular tray with straight sides. The base area of the tray is *A* and its height is *h*. What is the depth, *d*, of the liquid in the tray?



Students should also be able to derive and uses formulas for lengths of arcs and areas of sectors and segments.

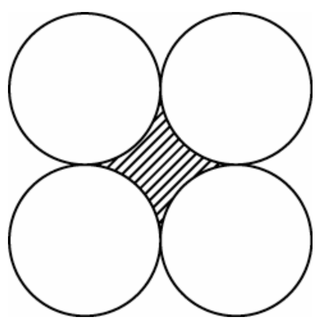
Here is an example of a problem that students should be able to solve:

Four Circles

The circles below each have a diameter of 5 inches.

What is the area of the shaded region?

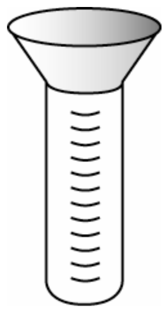
(Give your answer first in terms of π, and then as a decimal to two significant figures.)



Measurement

In general students will be expected to apply their knowledge of mathematics to solve measurement problems such as the following:

Rain Gauge Cylinder



Some rain gauges have a funnel shaped collector at the top of the cylinder so that rainfall from a wider area can be collected.

In the rain gauge pictured at the left, the radius of the top of the funnel is 4 cm,. The radius of the cylinder below it is 2 cm., and the height of the cylinder is 12 cm.

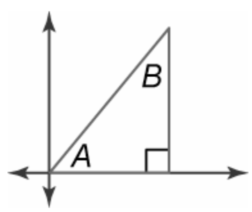
Suppose that in a heavy rain the bottom cylinder is filled to the top. How much rain has fallen? Explain how you know.

Specifically, students should have the opportunity to understand why radian measure is useful and be given the opportunity to convert between radian measure of angles and degrees.

Here is an example:

Angle Measure

In the triangle below, $m\angle A = 50^{o}$



What is the radian measure of angle *A*? Express your answer in terms of π.

Coordinate Geometry

Students should have the opportunity to solve specific problems using analytic geometry (including problems in three dimensions) and circular trigonometry (e.g. find the equation of a circle inscribed in a triangle given the coordinates of the vertices).

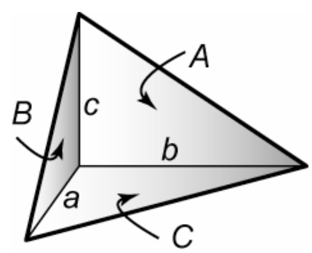
Here is an example of a problem that prospective mathematic major should be able to solve:

Convex Pentagon

In a convex pentagon *ABCDE,* the sides have lengths 1, 2, 3, 4, and 5, though not necessarily in that order. Let *F*, *G*, *H*, and *I* be the midpoints of the sides *AB*, *BC*, *CD*, and *DE*, respectively. Let *X* be the midpoint of segment *FH*, and *Y* be the midpoint of segment *GI*. The length of segment *XY* is an integer. Find all possible values for the length of side *AE*.

The Pythagorean Theorem in Three Dimensions

In a rectangular box, a "body diagonal" is a line segment from one corner through the interior of the box to the opposite corner.



Find the length of the body diagonal of a rectangular box in terms of the lengths *a*, *b*, and *c* of the sides of the box.

A "body triangle" of a rectangular box is a triangular region inside the box formed by a plane that passes through three face diagonals of the box.

Each body triangle defines a tetrahedron formed by the body triangle itself and the three triangular half faces of the box that meet at the vertex opposite the body plane. See the diagram.

A, *B*, and *C* are the areas of the three right triangles. *D* is the area of the shaded triangular face of the tetrahedron.

Show that the sum of the squares of the areas *A*, *B*, and *C* of the triangular half faces is equal to the square of the area of the body triangle.

(Hint: Use Heron's Theorem to find the area of the body triangle. The proof is straightforward, but it does require some algebraic manipulation.)