Equivalent Expressions Using the Distributive Property

Grade and Content Area	Grade 8 Mathematics
Title	Equivalent Expressions Using The Distributive Property
GLEs/GSEs	M(F&A)-8-4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., $d = rt$; $d/r = t$); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context.
Context of the Lesson	Although students can verify the equivalence of expressions by reasoning about the context or by comparing graphs and tables (as done in previous lessons), it is important that they develop methods for showing equivalence by using symbolic reasoning. In this lesson, students use their prior knowledge of area of squares and rectangles to calculate two methods for finding the total surface area of the water on top of a pool. The distributive property emerges naturally as students analyze swimming pools that are divided into sections and find various methods for computing the surface area of the water. (Lappan, et. al., p. 1c) Lesson adapted from Investigation 2, Say It With Symbols, *Connected Mathematics Project*.
Opportunities to Learn	**Materials** • Worksheets – includes diagrams and questions about the problem “Diving In” • An overhead projector • The “Diving In” problem on an overhead transparency • Algebra tiles are used as a hands-on tool for students to better visual the problem.

Opportunities to Learn Continued

Professional Resources and References:

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2004). Say It With Symbols Teacher's Guide. In *Connected Mathematics Project.* Glenview, IL: Pearson Prentice Hall.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2004). Say It With Symbols Student Edition. In *Connected Mathematics Project.* Glenview, IL: Pearson Prentice Hall.

Class organization

- Engage all students' participation through whole class discussions.
- Students work in pairs or small groups and report out results to large group.
- Teacher facilitates large group reports and employs questioning techniques to encourage participation.

Classroom Climate

- Encourage students to work and talk with their partner as they engage in the task.
- The classroom is a safe place to take risks and try new methods of solving problems.
- The teacher models respect for all learners, all questions, and all methods, and encourages careful listening at all times.

Differentiated Learning

- Pair students who exhibit varying levels of understanding with careful consideration to ensure that both students benefit from this arrangement.
- The worksheets that students receive will be the same problem but with varying degrees of difficulty. All students will be asked to complete Parts A and B on the worksheet, which deals with equivalent numerical expressions.
- If groups finish early, they will extend their knowledge to a more difficult application of the same problem, using algebraic expressions.

The overview to the *Connected Mathematics Program* Special Needs Handbook available on the Prentice-Hall Web site http://www.phschool.com/cmp2/pdfs/snh_overview.pdf provides many suggestions for addressing the needs of special needs students.

Depth of Knowledge

Level 3

	This activity requires students to reason and plan a sequence of steps. However, the first three depth of knowledge stages (recall, skills/concepts, and strategic thinking) will be incorporated throughout the lesson. The problem contains some complexity and more than one possible answer. The students need to show both ways to solve this problem. The problem requires an informal conjecture of the distributive property and a restructuring of the problem to show equivalent expressions.
Objective(s)	**Students will be able to do the following** 1. Articulate the distributive property independent of a specific context (informally) 2. Further articulate what it means for two expressions to be equivalent 3. Write two equivalent expressions in factored form and expanded form of the distributive property
Instructional Procedures	**Opening** (5 – 10 minutes) 1. Begin the lesson by writing on the overhead the following expressions: a. s + s + s + s + 4; b. 4s + 4. 2. Tell the students to look carefully at these algebraic expressions. Ask them the following questions: a. "Are the two expressions equivalent?" b. "How could you convince a student that the two expressions are equivalent?" 3. Depending on the level of understanding, the students may note that s + s + s + s + 4 is four s's combined to give the expression 4s + 4. 4. Next, write on the overhead the following expressions: a. 4s + 4; b. 4(s + 1) 5. Tell the students to look carefully at these algebraic expressions. Ask them the following questions: a. "Are the two expressions equivalent?" "How could you convince a student that the two expressions are equivalent?" 6. If students have trouble with this equivalence, guide them with the order of operations, i.e., parentheses first. 7. Draw a rectangle on the overhead with labels L for length and W for width. Tell the students that one person wrote the perimeter as 2L + 2W and another person wrote the perimeter as 2(L + W). Ask them how could you convince the students that these two expressions are equivalent?" The students may note that there are two lengths and two widths for both expressions. 8. Point out that in today's problem, the students should be on the lookout

Instructional Procedures Continued	for relationships among the equivalent expressions that they will write. 9. Put up the ditto "Diving In" on the overhead. Discuss the worksheet "Diving In" with students. Talk about how swimming pools are sometimes divided into sections that are used for different purposes. Ask students for different reasons a swimming pool would be divided into sections. 10. Put the following on the board or have it previously written: Summary: On the back of your worksheet, write a summary using the areas of rectangles to explain why your two expressions are equivalent. 11. Pass out the worksheet "Diving In".
	Engagement (25 – 30 minutes) 1. Have students work individually for 2 – 5 minutes to give all students a chance to discover the multiple ways to express the area of the pool. Give the students time to think about the problem. 2. Letting the students work alone for a few minutes will ensure that another student's strategy does not prohibit his/her partner's way of thinking about the problem. 3. Once students have had time to think about the problem, have students work with a partner to share and discuss their ideas. 4. Circulate the room to see what each pair of students is discussing. 5. To help troubled students, ask them the following questions: a. "What is the area of the left side of the pool, the shallow end?" b. "What is the area of the right side of the pool, the deep end?" c. "What is the area of the entire pool without the split sections?" 6. If difficulty still arises, ask the following questions: a. "What is the length of this rectangle?" b. "What is the width of this rectangle?" 7. For pool A, be sure that students are showing two methods and not just giving a numerical answer. It is important for students to write the numerical expressions down on paper, and then solve using the order of operations. 8. Be sure students can informally articulate the concept of the distributive property. a. Ask students the following questions for pool A: b. "How did you arrive at your area of 2100 square meters?" c. "Can you write down the steps you took and show all your work?" d. "Is it possible to write one expression that shows all the operations needed to calculate the total surface area of the pool?" 9. Continue to ask questions of all students to be sure the students can informally articulate the concept of the distributive property. 10. Continue to walk around the room to each pair of students asking the same questions.

Instructional Procedures Continued	**Closure** (10 – 15 minutes) 1. Ask students to share with the class their summarized methods that they used to calculate the total surface area of the water of the pool. They should suggest that you could find the surface area of each section by multiplying the length by the width, and add the two products. You could also add the lengths of the two sections, and multiply that sum by the common width. 2. Ask a student to show the two methods for calculating the area for pool A. 3. Ask another student to show the method in finding the total area for pool B and, then, pool C. 4. Each time a student shows his/her work on the overhead, be sure to ask them to summarize how they decided on each expression that they wrote. Continue to ask the students how they know the two expressions are equivalent. 5. Pass out three additional problems involving the area of rectangle. Tell the students that this is “their ticket out of class” and that this will check their understanding of today’s lesson. Have the students work alone on these problems.
Assessment	• Teacher observation of student participation during class discussion and group work. • Student worksheet complete for “Diving In.” This includes a written summary explaining why their two expressions are equivalent. • This assessment is informative; however, further assessment of this GLE will involve individual student written responses. Some students will have an opportunity to add their verbal understanding of the distributive property. • Students’ work on the “three additional problems” will enable the teacher to know how much further guidance is needed on this particular topic.
Reflection	**Student Work Sample 1: Approaching Proficiency** The student who is approaching proficiency explained that the two expressions are equivalent by saying that the length times the width plus the length times the width equals the length plus the length and then multiplied by the width. They stated this equivalency without explaining why it makes sense that the two expressions are equivalent. They did not use the areas of the pools to help them with their explanations. They also lacked the understanding that the two lengths may or may not be the same value.
	Student Work Sample 2: Proficient The students who are proficient explain why the two expressions are equivalent using pool A as an example. They show that both expressions are equivalent by explaining the order of operations to get the same numerical answer.

Reflection Continued	**Student Work Sample 3: Exceeds Proficiency** The students who exceed proficiency explain why the two expressions are equivalent using the areas of rectangles to explain their understanding. They explain that the area of each section of the pool added together is equivalent to the area of the entire pool with one combined length. They generalize the problem when there was an unknown length. The student uses either pool B or pool C as an example to help with their explanation. A rare case of a student who exceeds proficiency is when the student explains why their two expressions are equivalent using algebraic manipulation of symbols. They can use the distributive property with or without using the name of the property to show their equivalency.
	Lesson Implementation The lesson objectives related to the GLE were met. The students wrote two equivalent expressions and discussed why they were in fact equivalent. The students were able to informally discuss why their two expressions were equivalent. They understood how to find the area of rectangles so many students took their prior knowledge of area to create two equivalent expressions. All students when working with pool A met the lesson objectives related to the GLE. All students with the help of their partner and my questioning could determine two methods for finding the area of the pool when there was a numerical answer. When a variable was given as one of the side lengths, many students assigned a value for that variable based on the relation to the other values in the problem. For example, if one side of the pool looked like a square, the students gave the variable x a value according to the square's length. In doing this, many students had a numerical answer for pool B and pool C instead of an algebraic expression to represent the pool's area.