

Development of Mathematics Expectations

The PK-16 Mathematics Advisory Committee is a working committee of the PK-16 Council chaired by Governor Donald Carcieri. The committee formed in March 2004 has met and communicated since that time to develop guidelines for college readiness in mathematics. The committee was chaired by Prof. Lewis Pakula of URI and co-chaired by Judith Keeley of RIDE, with representatives from RIC, CCRI, RIDE, and a number of Rhode Island high schools. (See full list below.) The committee agreed on the following principles about students' expectations, excerpted from the Executive Summary of the committee's final report:

All students who expect to take and succeed in entry-level courses at the university/college level should be proficient in what was once understood as Algebra II. In particular, they should be able to solve problems in arithmetic and algebra, particularly those involving fractions in any form, using conventional notation and algorithms. Such problems should entail appropriate and correct use of arithmetic and algebraic methods and reasoning, and the correct systematic computation of the answer. Students should be able to do small-scale problems without use of a calculator and be able to make appropriate use of a calculator and other technology in applications. Students should have the widest possible experience solving problems involving application of mathematics, as well as problems that call for careful and extended reasoning about mathematics itself.

All students should have facility with geometric reasoning. Among other things, they should know the formulas for area, perimeter and volume related to basic geometric objects and be able to use these to solve problems involving more complicated objects. They should be able to reason about similar and congruent triangles and know the Pythagorean Theorem, its applications and related trigonometry, and, ideally, a proof. They should be able to form inverses, converses and contrapositives of geometric (and more general) statements, give definitions of geometric terms, and make geometric deductions.

Topics in discrete mathematics and probability that reinforce basic competence in algebra and arithmetic are valuable for students intending to take general education courses at the college level.

While geometry, statistics, discrete mathematics, and other fields are obviously important, the strong consensus among mathematicians at the college level is that inadequate mastery of arithmetic and algebra is the greatest impediment to success in college mathematics, whether for general education or in mathematically intensive courses of study. Those curricular reforms in recent years that encourage more problem solving, student engagement, and lively content are laudable and should complement the rigorous development of the basics for college readiness.

These content expectations are intended to be consistent with the RI GSE's 11-12 as developed. Further details will be found below.

PK-16 Mathematics Advisory Committee

Joseph Allen, Department of Mathematics, CCRI
Peter Andreozzi, RIDE

Donna Christy, Department of Mathematics, RIC
Patricia Dulac, East Greenwich High School
Barbara Fox, RIDE
David Heskett, Department of Physics, URI
Judith Keeley, RIDE, Co-chair
Paula Najarian, Tolman High School
Ann Moskol, Department of Mathematics, RIC
Lewis Pakula, Department of Mathematics, URI, Chair
Diane Schaefer, RIDE
Lois Short, Burrillville High School

Mathematics Readiness without Remediation

Students should enter college prepared for success and equipped to make choices of major and career based on their interests, talents, and aspirations, and not limited by their mathematics background. For example, a student who may need only a single finite mathematics course for her initial major choice in psychology should be able to switch to a precalculus/calculus track if her interests change to marine biology, without remedial coursework. Thus, these recommendations apply, in principle, to all students entering four year college programs or who intend to transfer to such programs from a community college.

For example, both URI and RIC have stated admissions requirements in mathematics of one year of geometry and two years of algebra. Interpretation of these requirements, consistent with their original intent and current expectations, is indicated below. Although students in general education mathematics courses might make active use of a different subset of the items below than those taking technically oriented courses, the core arithmetic/algebraic skills, and experiences with mathematical reasoning, problem solving and presentation, are relevant to all students.

There is wide agreement that an informed fluency with the algorithms and methods of arithmetic and basic algebra, based on principles and reasoning, is a cognitive and conceptual precursor to more advanced mathematics.

Basic Skill and Knowledge Expectations

Students should

- Solve a wide variety of problems applying basic knowledge and techniques (see below) at various scales of difficulty and complexity: simple calculations, modeling problems where verbally expressed relationships must be put in algebraic form and deductions made, and problems involving the interplay between graphical and algebraic perspectives.
- Approach problems multi-representationally, using algebraic, graphical, numerical, and verbal perspectives, and effectively using technology when appropriate.

- Present logical reasoning and proofs in various mathematical contexts (e.g. algebra, geometry, modeling), and provide careful explanations of problem formulations, solutions, and procedures when required.

Students should also acquire specific skills and knowledge of mathematics. The items below are not intended to be exhaustive, but only to indicate the scope and level of proficiency implicit in the Algebra II and Geometry expectations. The items do not include everything about the mathematical sciences that would constitute an exemplary high-school program, such as basic statistics, additional topics in geometry, and computing.

Students should be able to

- Use correct conventional notation for arithmetic and algebraic expressions and formulas, including parentheses, order of operations, equality and inequality signs.
- Understand the real numbers as decimal expansions, represent them as points on a line, graphically interpret order relations for real numbers, and perform order and absolute value comparisons using arithmetic.
- Perform, reliably by hand, all the basic operations of arithmetic, including division, especially as applied to fractions. Convert between ordinary fractions, decimals and percents, by hand in simple cases, and with calculators in more complicated ones. Use calculators to supplement and leverage skill in hand calculation when appropriate.
- Interpret and compute with integer and fractional exponents, radicals, and scientific notation.
- Compute and simplify sums, differences, and products of general polynomials.
- Model linear relationships both algebraically in conventional forms, and graphically, interpret coefficients in graphical terms, e.g. as slopes and intercepts, and solve problems relating these representations to each other.
- Solve quadratic equations by factoring, completing the square, and the quadratic formula, and use these techniques to solve applied problems, and obtain a rough hand sketch of the graph of a quadratic equation from its algebraic expression.
- Solve equations that can be reduced to linear or quadratic equations by hand and use a graphing calculator to estimate solutions of equations not feasibly solvable by hand. Solve simple systems of equations.
- Solve inequalities and use inequalities and absolute value to specify constraints on sets of numbers.

- Add, multiply, and divide rational expressions, that is algebraic fractions, and reduce them to lowest terms, simplify compound fractions, and perform simple polynomial division.
- Understand the function concept and the use of function notation, know how functions can be represented by graphs, draw inferences from the graphs about the function (values, maximum values, increasing/decreasing, solutions of related equations, etc.).
- Solve elementary counting problems involving permutations and combinations and apply these to probability calculations.
- Use the definition and properties of common and natural logarithms to solve simple exponential equations.
- Solve computational and reasoning problems about congruent and similar triangles, especially those involving proportions, problems in the geometry of circles, and problems involving the Pythagorean Theorem.
- Know, for automatic recall, the formulas for the areas and perimeters of triangles, rectangles and circles; angle relationships in triangles and polygons; and the volumes of simple solids. Be able to apply these to the measurement of more complicated geometric objects.
- Be able to form inverses, converses and contrapositives of implications expressing geometric (and other) propositions, give definitions of geometric terms, make valid geometric deductions with explanations, and give coherent proofs of geometric propositions.
- Know the properties of special triangles (right, equilateral, isosceles right), the common angles (30, 45, 60, 90 degrees) especially their sines and cosines.
- Know the definitions of the basic trigonometric functions, calculate them by hand or by using a calculator as appropriate, solve right triangles, and use these techniques in simple applications.

Readiness for Technical/Scientific Programs

Students who intend to pursue studies in mathematical sciences, natural sciences, applied sciences, business or engineering should have substantial additional training in precalculus mathematics, including trigonometry and more advanced topics in algebra and geometry, and should be prepared to start calculus upon college entry. They should be strongly advised to achieve this additional proficiency while in high school.

In addition to the items above, such students should, for example,

- Perform algebraic operations on functions; determine domains, ranges, compositions/decompositions and inverses. Reason about functions not representable by algebraic formulas, and use functions as models.
- Reason about functions from graphical and algebraic representations, relate algebraic forms to graphical features, (e.g., scaling and location transformations) using graphing calculators only to supplement conventional analysis.
- Know systematically, and use, the basic families of elementary functions: linear and quadratic, polynomial, rational, logarithmic and exponential, and trigonometric functions. Employ confidently the formal properties and algebraic techniques associated with each family, be able to relate the parameters of each family to corresponding graphs, and use these functions in applications.
- Know additional topics in algebra, e.g. the arithmetic of complex numbers and how they arise in solving polynomial equations, basic theory of equations, how to solve more involved algebraic equations and systems of equations, how to sum finite and infinite geometric series.
- Know and be able to solve problems in trigonometry, including solution of triangles, verification of trigonometric identities, especially Pythagorean, period, and sum/difference identities, the geometry of complex number arithmetic, and a variety of applications.
- Know and be able to solve problems in analytic geometry, including conic sections.
- Acquire greater facility with geometry by, for example, proving less obvious propositions in geometry using synthetic and analytic methods, devising and validating constructions, expressing symmetry, congruence and similarity relations in terms of transformations, solving problems in solid geometry.

Success in calculus depends critically on a strong background in *precalculus*.

Continuity of Mathematics Experience

All students should have a substantial course in the mathematical sciences in their senior year, and should be encouraged to take the most intensive mathematical course for which they are ready. Students well prepared in precalculus should take Advanced Placement calculus in their senior year if they intend to pursue a scientific or technical major in college.

General Academic Readiness

In addition to the skill and knowledge expectations indicated in the items above, students should have developed productive attitudes and a growing maturity with respect to the study of mathematics. They should

- Be able to persevere with more challenging problems and to recognize that mastery requires practice, especially through diligent effort with homework.
- Have seen motivating and enlivening real-world applications of mathematics, but students should understand that not every essential topic in high school mathematics can be so motivated, and that some topics are to be studied because they provide background for deeper skills and understanding to be gained later. Students should see mathematics as a major collective human achievement, empowering each new user with its accumulated store of knowledge and technique.
- Appreciate the need to learn, for automatic recall, basic number facts, definitions, theorems, formulas, techniques and algorithms, so that these basics will be freely available to them when solving problems or encountering new mathematical ideas.
- Have acquired reliable habits of mathematical work and communication, including precise use of language and conventional notation, and competent recording of symbolic manipulations.