

# DISCOVERING THE DEFINITION OF ROTATION

*A Guided Investigation*

## **Geometry-Congruence**

### **Experiment with Transformations in the Plane**

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## **Teacher Notes.**

**Prior knowledge.** It is suggested that students have prior knowledge of a parts of a circle (e.g., center, radius, and diameter) and prior experience performing rotations with protractor and compass before attempting this activity. Students should also be proficient with a protractor and a compass as well as familiar with terms such as distinct and unique (or unambiguous).

**Tools.** protractor and compass per student

**Contents.** Contained within are:

- one example illustrating the final work product of using a protractor and compass to graph the image of a geometric figure after a rotation of certain degree around the origin,
- prompts for which students must add the background protractor and compass work, and
- prompts for which students must carry out the rotation in its entirety.

**Implementation.** As this activity is designed as a guided investigation, students with the prerequisite knowledge and skills should be able to complete the activity without much assistance. The exercises can be time consuming for some students, so you might want to assign some or all of the exercises as homework in lieu of allocating class time. It is suggested that students be encouraged to make their conclusions in collaboration with other each other

and that the collaboration period be followed up by a whole-class discussion to enhance their understanding of the definition of rotation.

*Please note that this file is being transmitted as a Word file so that you may manipulate its components to best fit the needs of your specific student populations.*

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## Part 1. Exercises

### Protocol

1. With your group members assume a student role (i.e., S1, S2, ...) based upon the number of students at your table.
2. Reference the work-distribution guide (the second page of this document) to determine which rotation exercises you are responsible for performing.
3. Perform the specified rotations.
4. When everyone has performed their rotations—and before proceeding to Part 2—take the time to compare your results with one or more students within your group who completed the same rotation. If discrepancies occur, please talk them through to resolution. By all means, invite me into the discussion if you are unable to resolve the discrepancy on your own.

## **Work-Distribution Guide**

### **Three-student table**

**S1: A, B1, B2, C2, C4, C5, C6**

**S2: A, B1, C1, C2, C3, C4, C5**

**S3: A, B1, B2, C1, C3, C4, C6**

### **Four-student table**

**S1: A, B1, C1, C4, C6**

**S2: A, B2, C2, C3, C5**

**S3: A, B1, C2, C4, C6**

**S4: A, B2, C1, C3, C5**

### **Five-student table**

**S1: A, B1, C1, C3**

**S2: A, B1, C2, C4, C6**

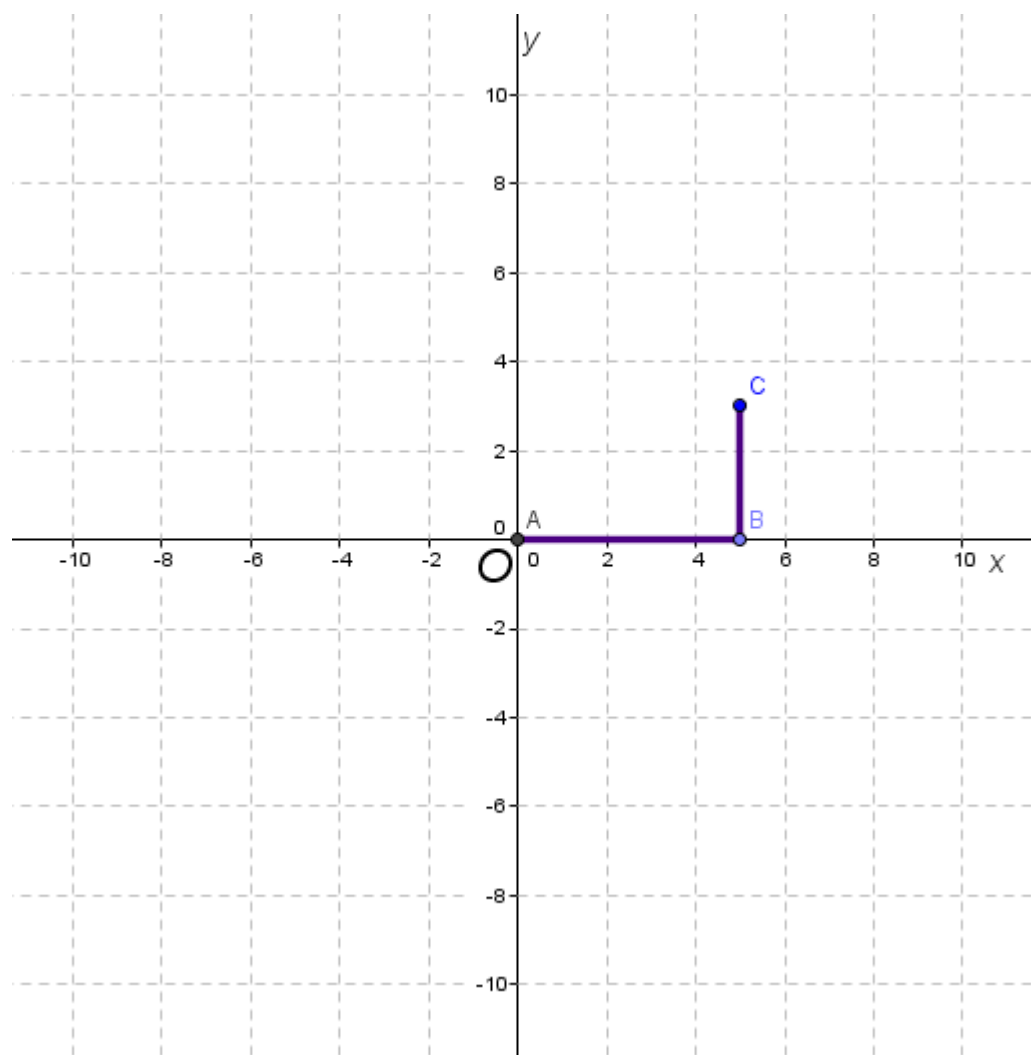
**S3: A, B1, C1, C5**

**S4: A, B2, C2, C4, C6**

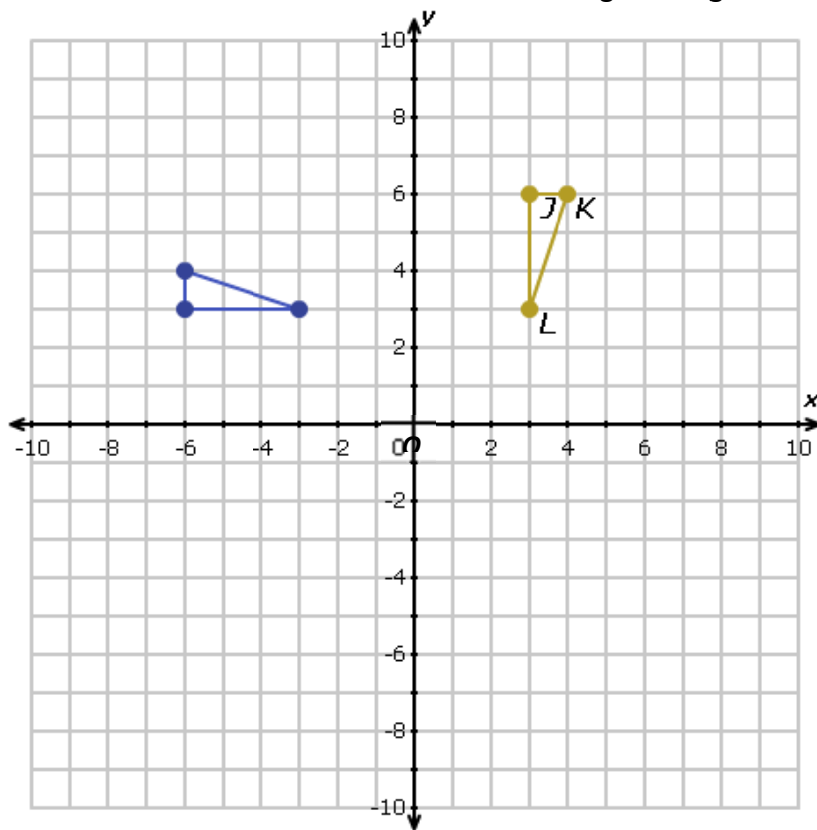
**S5: A, B2, C3, C5**

### Exercise A. You Try It.

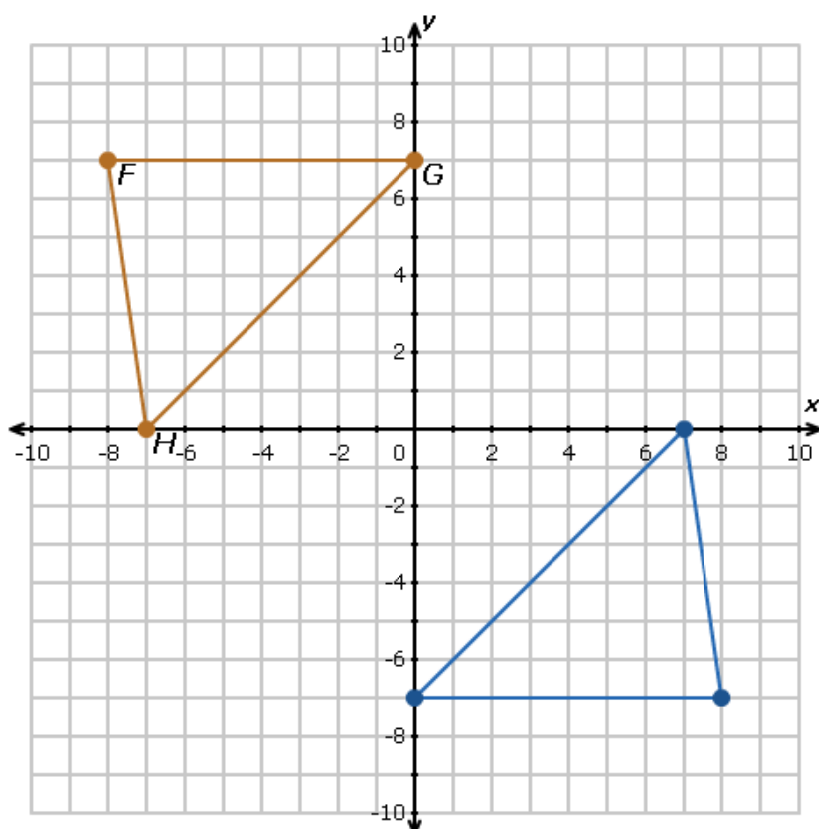
**Directions.** Sketch the image of the figure L after it is rotated  $60^\circ$  counterclockwise around point  $O$  (optionally denoted  $Ro_{O, 60^\circ}$ ).



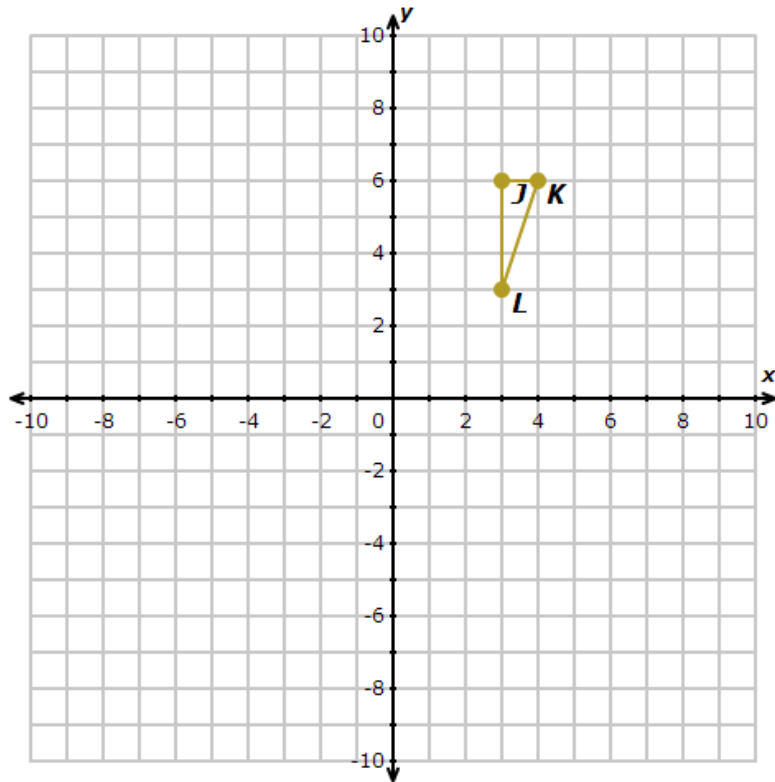
**B.1. Directions.** Use a protractor and compass to determine the placement of the image of  $\triangle JKL$  after a  $90^\circ$  counterclockwise rotation using the origin as the center of rotation.



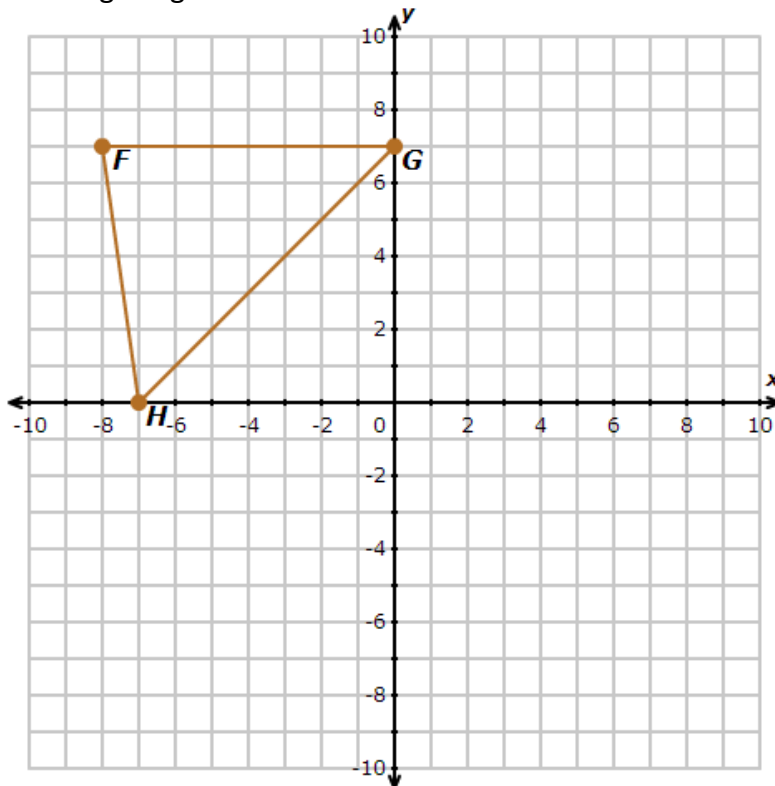
**B.2. Directions.** Use a protractor and compass to support the placement of the image of  $\triangle FGH$  after a  $180^\circ$  rotation using the origin as the center of rotation.



**C.1. Directions.** Use a protractor and compass to graph the image of  $\triangle JKL$  after a  $45^\circ$  counterclockwise rotation using the origin as the center of rotation.



**C.2. Directions.** Use a protractor and compass to graph the image of  $\triangle FGH$  after a  $180^\circ$  rotation using the origin as the center of rotation. Please record the coordinates of the vertices of the resulting image.

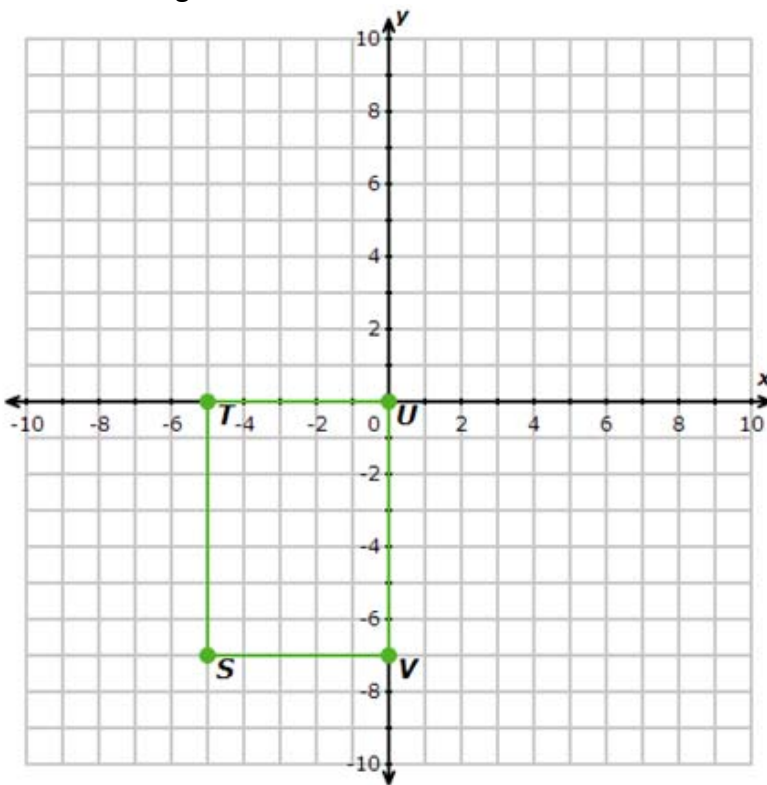


$$F(-8, 7) \rightarrow F'(\underline{\quad}, \underline{\quad})$$

$$G(0, 7) \rightarrow G'(\underline{\quad}, \underline{\quad})$$

$$H(-7, 0) \rightarrow H'(\underline{\quad}, \underline{\quad})$$

**C.3. Directions.** Use a protractor and compass to graph the image of  $TUVS$  after a  $90^\circ$  rotation around the origin. Record the coordinates of the vertices of the resulting image.



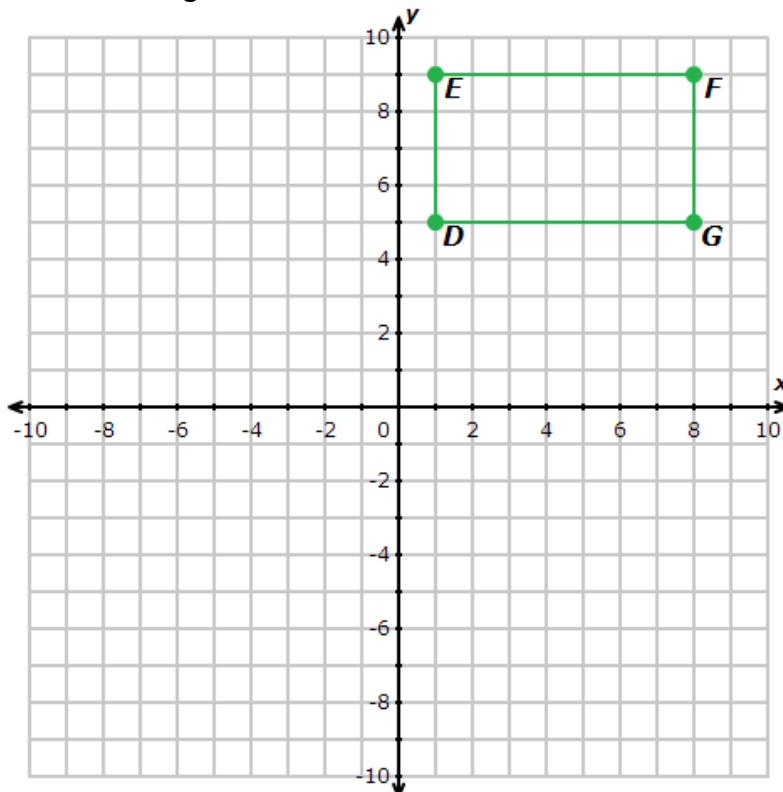
$$T(-5, 0) \rightarrow T'(\underline{\quad}, \underline{\quad})$$

$$U(0, 0) \rightarrow U'(\underline{\quad}, \underline{\quad})$$

$$V(0, -7) \rightarrow V'(\underline{\quad}, \underline{\quad})$$

$$S(-5, -7) \rightarrow S'(\underline{\quad}, \underline{\quad})$$

**C.4. Directions.** Use a protractor and compass to graph the image of  $EFGH$  after a  $0^\circ$  rotation around the origin. Record the coordinates of the vertices of the resulting image.



$$E(1, 9) \rightarrow E'(\underline{\quad}, \underline{\quad})$$

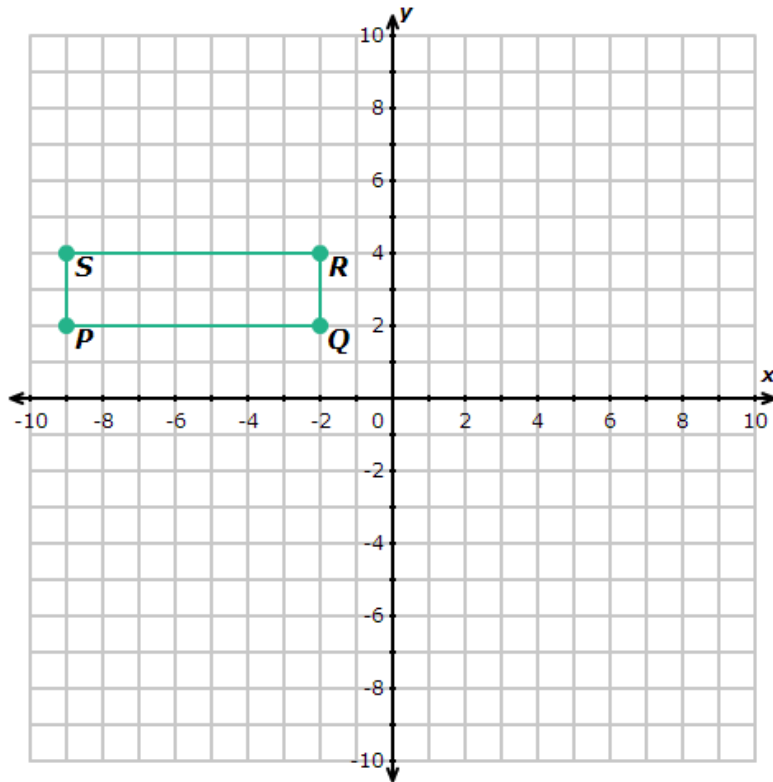
$$F(8, 9) \rightarrow F'(\underline{\quad}, \underline{\quad})$$

$$G(8, 5) \rightarrow G'(\underline{\quad}, \underline{\quad})$$

$$D(1, 5) \rightarrow D'(\underline{\quad}, \underline{\quad})$$



**C.5. Directions.** Use a protractor and compass to graph the image of  $SRQP$  after a  $-90^\circ$  rotation around the origin. Record the coordinates of the vertices of the resulting image.



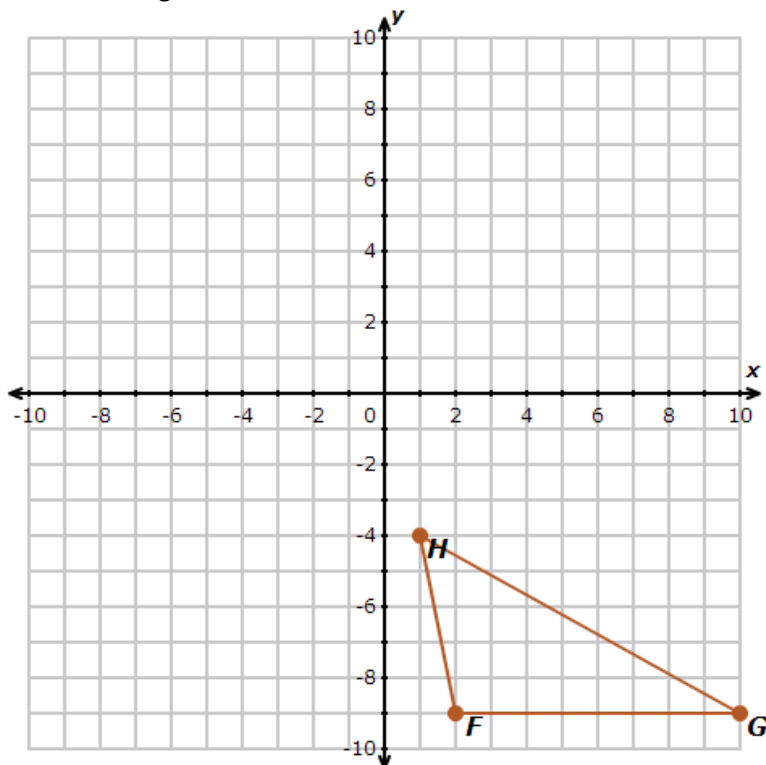
$$S(-9, 4) \rightarrow S'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$R(-2, 4) \rightarrow R'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$Q(-2, 2) \rightarrow Q'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$P(-9, 2) \rightarrow P'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

**C.6. Directions.** Use a protractor and compass to graph the image of  $\triangle HGF$  after a  $0^\circ$  rotation around the origin. Record the coordinates of the vertices of the resulting image.



$$H(1, -4) \rightarrow H'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$G(10, -9) \rightarrow G'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

$$F(2, -9) \rightarrow F'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

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## **Part 2. Make Conclusions**

Group members will need to work together in order to answer the questions that will inform a precise definition of Rotation.

## Make Conclusions Based Upon Your Work.

For definiteness, we first deal with the case where  $0 \leq t \leq 180$ .

$P$  is any point of the pre-image.

Look specifically at exercises A and C.3. What is the location of points  $A$  and  $U$ , respectively, relative to the origin?

What is the location of  $A'$  and  $U'$ , respectively, again relative to the origin?

What is the observed effect of rotating such a point, for any value of  $t^\circ$ , around the origin?

Based upon these two observations, complete the general rule.

If  $P = O$ , then by definition,  $Ro(O) = \underline{\hspace{1cm}}$ .

Look at any points in any of the exercises other than points  $A$  and  $U$  just considered, are the points distinct or non-distinct from the origin? Why?

When rotated, where are the images in relation to their respective pre-images?

In contrast to the preceding rule,

### **MULTI FILL-IN-THE-BLANK**

If  $P$  is distinct from  $O$ , then by definition,  $Ro(P)$  is the point  $Q$  on the

(1) \_\_\_\_\_ with (2) \_\_\_\_\_  $O$  and (3) \_\_\_\_\_  $|OP|$  such

that  $|m\angle QOP| = t^\circ$  and such that  $Q$  is in the

(4) \_\_\_\_\_ direction of the point  $P$ .

circle / radius / diameter / angle / degree / clockwise / counterclockwise / center

Looking at any point of any geometric image rotated in this investigation, does the mapping result in a single image or multiple images?

### **MULTIPLE CHOICE**

Thus, one may claim that the mapping is \_\_\_\_\_ (i.e., there cannot be more than one such image  $Q$ ).

unique / ambiguous / congruent / unambiguous

Look at exercises B.2 and C.2, where geometric figures are rotated  $180^\circ$  around the origin, if each pair of pre-image/image points (e.g.,  $H$  and  $H'$ ) are treated as endpoints of a segment, what special chord of a circle does each pre-image/image pair create?

**MULTIPLE CHOICE**

If  $t = 180$ , then  $Q$  is the point on the circle so that  $\overline{PQ}$  is a \_\_\_\_\_ of the circle.

secant / diameter /

Look at exercises C.4 and C.6, where geometric figures are rotated  $0^\circ$  around the origin, describe the result of the mapping.

**MULTIPLE CHOICE**

If  $t = 0$ , then  $Q = P$ ; and  $R_0$  is the \_\_\_\_\_ transformation of the plane.

reciprocal / identity / non-rigid / inverse

### MULTIPLE CHOICE

Hence, if  $0 < t < 180$ , then all the  $Q$ 's in the

\_\_\_\_\_ direction of the point  $P$  with the

property  $0 < |m\angle QOP| < 180^\circ$  lie in the fixed half-plane of  $\overrightarrow{OP}$  that contains  $Q$ .

clockwise / counterclockwise / circumference

Thus,  $Ro$  is well-defined, in the sense that the mapping is ***unambiguous***.

Now, consider the case when  $t < 0$ .

### MULTIPLE FILL-IN-THE-BLANK

By definition, we rotate the given point  $P$  \_\_\_\_\_ on the circle

that is \_\_\_\_\_ at  $O$  with \_\_\_\_\_  $|OP|$ . Everything

remains the same except that the point  $Q$  is now the point on the circle so that

$|m\angle QOP| = |t|^\circ$  and  $Q$  is in the \_\_\_\_\_ direction of  $P$ .

circle / radius / diameter / angle / degrees / clockwise / counterclockwise / center

Thus, we define  $Ro(P) = Q$ .